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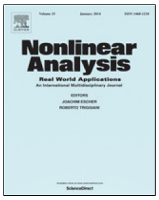

Hopf bifurcation in a reaction–diffusion model with Degn–Harrison reaction scheme[☆]

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ABSTRACT

In this paper, we consider a chemical reaction–diffusion model with Degn–Harrison reaction scheme under homogeneous Neumann boundary conditions. The existence of Hopf bifurcation to ordinary differential equation (ODE) and partial differential equation (PDE) models are derived, respectively. Furthermore, by using the center manifold theory and the normal form method, we establish the bifurcation direction and stability of periodic solutions. Finally, some numerical simulations are shown to support the analytical results, and to reveal new phenomenon on the Hopf bifurcation.

1. Introduction

In 1952, Turing predicted the spontaneous formation of chemical spatial structure from an initially uniform state in his landmark paper [1]. Later, the spontaneous formation was observed in the 1990s in controlled laboratory experiments [2,3]. Now, Turing mechanism has been extensively accepted and studied in diverse disciplines. Moreover, Turing's work has become an important approach to modeling spatial structures in ecological systems and chemical reactions, from patterns on animal coats to cluster formation in bacterial cultures [4]. Although the existence of Turing patterns is still a controversial issue in biology, it has been observed in chemistry (see [5–14] and the references therein).

In this paper, we are concerned with a chemical model introduced by Degn and Harrison in [15]. It is used to explain the observed oscillatory behavior of respiration rate in the continuous cultures of the bacteria

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Klebsiella aerogenes, and follows the form of the three-step reaction scheme:

$$\begin{aligned} A &\longrightarrow Y, \\ B &\Longleftrightarrow X, \\ X+Y &\longrightarrow P, \end{aligned}$$

where X and Y are the intermediate reactants, and represent oxygen and nutrient respectively; A and B explain "sources" or external parameters whose concentrations are to be kept at a constant level all over the reactor vessel; P is the final product whose concentration is also assumed to be constant. In the reaction process, the last step is considered to be inhibited by excess of oxygen in the reactor. The first and last steps are assumed to be irreversible, however the second step is reversible. For more background on this reaction scheme, one can refer to [15–18].

In [15], Degn and Harrison first proposed that the last step followed a nonlinear rate equation of the type $XY/1+qX^2$, where q measures the strength of the inhibitory law. With the homogeneous Neumann boundary conditions, the above Degn–Harrison reaction scheme is given by

$$\begin{cases} X_t - D_X\Delta X = k_2 B - k_3 X - \dfrac{k_4 XY}{1+qX^2}, & x\in\Omega,\ t>0, \\ Y_t - D_Y\Delta Y = k_1 A - \dfrac{k_4 XY}{1+qX^2}, & x\in\Omega,\ t>0, \\ \dfrac{\partial X}{\partial n} = \dfrac{\partial Y}{\partial n} = 0, & x\in\partial\Omega,\ t>0, \end{cases} \tag{1.1}$$

where A, B, X and Y denote dimensionless concentrations of the reactants; the constants k_i $(i=1,2,3,4)$ are reaction rates; D_X and D_Y, respectively, denote the Fickian molecular diffusion coefficients of X and Y, and they are assumed to be positive constants all over the reactor vessel. The rate and diffusion constants are parameters characteristic for a given system, and the concentrations A and B are variable parameters which can be controlled in the reaction process. For the detailed background of (1.1), one can refer to [15–17, 19,20,18,21–24].

So far, system (1.1) has received considerable attention; see, for example, [19,20,18,21–24,16,17]. These papers except for [16,17] focus either on the corresponding ordinary differential equation (ODE) system or on the reaction–diffusion system (1.1) in one-dimensional domain case. In [16], the authors studied the global attractivity of constant steady state, given some nonexistence results by considering the effect of the parameters D_Y and q respectively, and established the existence of nonconstant steady states by the degree theory. Finally, they discussed the Hopf and steady state bifurcations from constant steady state, but they do not obtain more information about Hopf and steady state bifurcations. Based on this observation, the authors [17] established the global structure of steady-state bifurcations from simple eigenvalues by bifurcation theory and the local structure of the steady-state bifurcations from double eigenvalues by the techniques of space decomposition and implicit function theorem, and these results complement and eventually sharpen the results obtained in [16]. Unfortunately, in [17], the authors still do not obtain more information about Hopf bifurcation.

Therefore, the main purpose of the present paper is to investigate the Hopf bifurcation of system (1.1). To simplify the system (1.1), we introduce the dimensionless quantities from [16]:

$$\begin{aligned} &\tau = k_3 t, \qquad u = \frac{X}{k_3}, \qquad v = \frac{Y}{k_3}, \qquad a = \frac{k_2}{k_3^2}B, \qquad b = \frac{k_1}{k_3^2}A, \\ &c = k_4, \qquad 1 = k_3^2 q, \qquad d_1 = \frac{1}{k_3}D_X, \qquad d_2 = \frac{1}{k_3}D_Y. \end{aligned}$$