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Global well-posedness of 3D magneto-micropolar fluid equations with mixed partial viscosity

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In this paper, we investigate the initial value problem for the three dimensional magneto-micropolar fluid equations with mixed partial viscosity. Global existence of smooth solutions are established by energy method, provided that the H^1 norm of the initial value is suitably small.

1. Introduction

The three-dimensional magneto-micropolar fluid equations have the following form

$$\begin{cases} \partial_t u - (\mu+\chi)\Delta u + u\cdot\nabla u - b\cdot\nabla b + \nabla\left(p + \dfrac{1}{2}|b|^2\right) - \chi\nabla\times v = 0, \\ \partial_t v - \gamma\Delta v - \kappa\nabla\nabla\cdot v + 2\chi v + u\cdot\nabla v - \chi\nabla\times u = 0, \\ \partial_t b - \nu\Delta b + u\cdot\nabla b - b\cdot\nabla u = 0, \\ \nabla\cdot u = 0, \qquad \nabla\cdot b = 0, \end{cases} \tag{1.1}$$

where $u = (u_1, u_2, u_3)$, $v = (v_1, v_2, v_3)$, $b = (b_1, b_2, b_3)$ and p are functions of $(x, y, z) \in \mathbb{R}^3$, $t > 0$, which denote the velocity of the fluid, the micro-rotational velocity, magnetic field and hydrostatic pressure, respectively. μ is the kinematic viscosity, χ is the vortex viscosity, γ and κ are spin viscosities, and $\frac{1}{\nu}$ is the magnetic Reynold.

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The incompressible magneto-micropolar fluid equations have attracted the attention of many physicists and mathematicians due to their important physical background, rich phenomena, mathematical complexity and challenges. The existence and uniqueness of local strong solutions was obtained by Galerkin method in [1]. In [2], the author proved global existence of strong solution with the small initial data. The existence of weak solutions and the uniqueness of the weak solutions in 2D case was established in [3]. Yuan [4] obtained local existence of smooth solutions and blow up criterion of smooth solutions. Later, various types of regularity criteria of weak solutions in different function spaces were established, we refer to [5–11]. Wang [7] investigated the initial value problem for the 3D magneto-micropolar fluid equations. Some new blow-up criteria of smooth solutions in terms of the vorticity and the velocity in a homogeneous Besov space are established. Cheng and Liu [12] studied the initial value problem for the 2D anisotropic magneto-micropolar fluid flows with mixed partial viscosity and established the global regularity of the 2D anisotropic magneto-micropolar fluid flows with vertical kinematic viscosity, horizontal magnetic diffusion and horizontal vortex viscosity. Mixed partial viscosity means the viscosity coefficients are different in different directions, even the viscosity coefficients disappear in some directions. Owing to mixed partial viscosity, some bad terms arise in the nonlinear terms and the viscosity terms fail in controlling the estimate of these bad terms.

If $v = 0$ and $\chi = 0$, then the magneto-micropolar fluid equations (1.1) reduce to MHD equations. The MHD equations govern the dynamics of the velocity and magnetic fields in electrically conducting fluids such as plasmas, liquid metals, salt water, etc. (see [13]). The local well-posedness of the MHD equations in the usual Sobolev spaces was established in [14]. But whether this unique local solution can exist globally is a challenging open problem in the mathematical fluid mechanics. Many researchers studied global well-posedness of the MHD equations and some interesting results were established, we may refer to [15–21]. Cao and Wu [15] obtained global regularity for the 2D MHD equations with mixed partial dissipation and magnetic diffusion by energy method. Lin, Xu and Zhang [17] also established the global well-posedness of classical solutions to the 2D MHD equations with zero diffusivity under the assumption that the initial velocity field and the displacement of the magnetic field from a non-zero constant are sufficiently small in appropriate Sobolev spaces. Lin and Zhang [18] proved the global well-posedness to 3D MHD-type equations by the energy method, which depends crucially on the divergence-free condition of the velocity field. The global well-posedness of smooth solutions to the 3D MHD equations with mixed partial dissipation and magnetic diffusion was proven by Wang and the second author [20]. Wang and Wang [21] proved global existence of solutions to the 3D MHD equations in the critical space χ^{-1}, which was introduced in [22] and used in studying the global well-posedness of the incompressible Navier–Stokes equations by Lei and Lin [23], provided that norm of initial norm of the initial value are bounded exactly by the minimal value of the viscosity coefficients. The space $\chi^{-m} = \left\{f : \left\||\xi|^{-m}\hat{f}\right\|_{L^1} < \infty\right\}(m \in \mathbb{R})$ is usually called the Wiener's algebra. Very recently, Lei [16] established global existence of classical solutions to the 3D MHD equations when the magnetic fields are purely swirling and perpendicular to the velocity fields.

In this paper, we investigate global existence of smooth solutions to 3D magneto-micropolar fluid equations with mixed partial viscosity

$$\begin{cases} \partial_t u - \mu_1 u_{xx} - \mu_2 u_{yy} - \mu_3 u_{zz} - \chi\Delta u + u\cdot\nabla u - b\cdot\nabla b + \nabla\left(p + \dfrac{1}{2}|b|^2\right) - \chi\nabla\times v = 0, \\ \partial_t v - \gamma_1 v_{xx} - \gamma_2 v_{yy} - \gamma_3 v_{zz} - \kappa\nabla\nabla\cdot v + 2\chi v + u\cdot\nabla v - \chi\nabla\times u = 0, \\ \partial_t b - \nu_1 b_{xx} - \nu_2 b_{yy} - \nu_3 b_{zz} + u\cdot\nabla b - b\cdot\nabla u = 0, \\ \nabla\cdot u = 0, \qquad \nabla\cdot b = 0, \end{cases} \tag{1.2}$$

with the initial value

$$t = 0: \qquad u = u_0(x,y,z), \qquad v = v_0(x,y,z), \qquad b = b_0(x,y,z). \tag{1.3}$$