Nonlinear Analysis: Real World Applications 27 (2016) 238–245

Contents lists available at ScienceDirect

Nonlinear Analysis: Real World Applications

www.elsevier.com/locate/nonrwa

Global stability of solutions to a free boundary problem of ductal carcinoma in situ☆

Shihe Xu

School of Mathematics and Statistics, Zhaoqing University, Zhaoqing, Guangdong 526061, PR China

ARTICLE INFO

Article history:
Received 13 February 2015
Received in revised form 2 August 2015
Accepted 5 August 2015
Available online 24 August 2015

Keywords:
Mathematical model
Ductal carcinoma in situ
Free boundary problem
Global stability

ABSTRACT

In the paper we present some remarks on the global stability of steady state solutions to a free boundary model studied by Xu (2004) and also prove some new results of global stability of steady state solutions to the model.

1. Introduction

The process of tumour growth and its dynamics is one of the most intensively studied processes in the recent years. There have appeared many papers devoted to develop mathematical models to describe the process, cf. [1–14] and references therein. Most of those models are based on the reaction diffusion equations and mass conservation law. Analysis of such mathematical models has drawn great interest, and many results have been established, cf. [15–25] and references therein.

Ductal carcinoma in situ (DCIS) refers to a specific diagnosis of cancer that is isolated within the breast duct, and has not spread to other parts of the breast. There are two categories of DCIS: non-comedo and comedo. Noncomedo type CDIS tends to be less aggressive. The most common non-comedo types of DCIS are: (1) Solid DCIS: cancer cells completely fill the affected breast ducts. (2) Cribriform DCIS: cancer cells do not completely fill the affected breast ducts; there are gaps between the cells. (3) Papillary DCIS: the cancer cells arrange themselves in a fern-like pattern within the affected breast ducts. (More information may be found in http://imaginis.com or [26].) In a talk Mary Edgerton described two special patterns found in DCIS vividly: one lining up like baby trees and one spreading out evenly with gaps.

☆ This work is partially supported by NSF of China (11301474, 11171295), Foundation for Distinguished Young Teachers in Higher Education of Guangdong, China (Yq2013163) and Foundation for Distinguished Young Talents in Higher Education of Guangdong, China (2014KQNCX223).

E-mail address: shihe56789@163.com.

http://dx.doi.org/10.1016/j.nonrwa.2015.08.003
1468-1218/

In [26], the author modifies a model proposed by Byrne and Chaplain [3], (also see Friedman and Reitich [27]), for the growth of a tumour consisting of live cells (nonnecrotic tumour) to describe the homogeneous growth inside a cylinder, a model mimicking the growth of a ductal carcinoma. The one dimensional model is in the form of a free boundary problem as follows.

$$c\frac{\partial \sigma}{\partial t} = \frac{\partial^2 \sigma}{\partial z^2} - \lambda\sigma, \quad \text{in } B(t),\ t > 0, \tag{1.1}$$

$$\frac{\partial \sigma}{\partial z}(0,t) = 0, \qquad \sigma(R(t),t) = \sigma_1, \tag{1.2}$$

$$\sigma(z,0) = \sigma_0(z), \quad \text{in } B(0), \tag{1.3}$$

$$\frac{\partial s}{\partial t} = \mu \int_0^{s(t)} (\sigma - \tilde{\sigma}) dz, \quad t > 0, \tag{1.4}$$

$$s(0) = z_0, \tag{1.5}$$

where $B(t) = \{z | 0 < z < s(t)\}$, $s(t)$ which represents the growing boundary of the tumour is unknown function; λ, μ are positive constants. $\lambda\sigma$ in (1.1) is the consumption rate of nutrient in a unit volume; σ_1 denotes the external concentration of nutrients, which is assumed to be constant. $c = T_{diffusion}/T_{growth}$ is the ratio of the nutrient diffusion time scale to the tumour growth time scale. Typically $c \ll 1$ (cf. [18,27,26]). The two terms on the righthand side of (1.4) are explained as follows: The first term $\mu \int_0^{s(t)} \sigma dz$ is the total volume increase in a unit time interval induced by cell proliferation, the proliferation rate is $\mu\sigma$; The second term $\mu \int_0^{s(t)} \tilde{\sigma} dz$ is the total volume decrease in a unit time interval caused by natural death, and the natural death rate is $\mu\tilde{\sigma}$. For details of the model, please see [26].

Global stability of steady state solutions to problem (1.1)–(1.5) were obtained in [26]. We realized that there are some mistakes in [26], which were reflected in Theorem 2.2 formulated in [26], so they were needed to be corrected. In presented paper we consider the global stability of solutions to one dimensional model studied in [26].

2. The limiting case when $c = 0$

First, we consider the limiting case when $c = 0$, i.e., we consider the following problem

$$\frac{\partial^2 \sigma}{\partial z^2} = \lambda\sigma, \quad \text{in } B(t),\ t > 0, \tag{2.1}$$

$$\frac{\partial \sigma}{\partial z}(0,t) = 0, \qquad \sigma(s(t),t) = \sigma_1, \tag{2.2}$$

$$\sigma(z,0) = \sigma_0(z), \quad \text{in } B(0), \tag{2.3}$$

$$\frac{\partial s}{\partial t} = \mu \int_0^{s(t)} (\sigma - \tilde{\sigma}) dz, \quad t > 0, \tag{2.4}$$

$$s(0) = z_0. \tag{2.5}$$

The solution to (2.1), (2.2) is

$$\sigma(z,t) = \frac{\sigma_1 \cosh(\sqrt{\lambda} z)}{\cosh \sqrt{\lambda} s(t)}. \tag{2.6}$$

Substituting (2.6) to (2.4), and setting $\sqrt{\lambda} s(t) = \eta(t)$, one can get

$$\eta'(t) = \mu\sigma_1 \tanh \eta - \mu\tilde{\sigma}\eta =: \mu f(\eta). \tag{2.7}$$

The specific value of μ will not affect the results of the model. We shall take $\mu = 1$ for simplicity in the following of the paper.