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Two dimensional spatial solitons in parity-time symmetric potential

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ABSTRACT

This work analyzes the propagation of (2 + 1) dimensional spatial solitons in parity-time ($\mathcal{PT}$) symmetric potential. The stationary solution of the system has been studied. The beam dynamics has been analyzed using variational and numerical methods. The soliton beam propagation is stable when the coefficient of imaginary potential is less than a threshold, which is called the phase transition point. Above the transition point, the imaginary component of the solution starts to evolve and the solution becomes unstable. When the coefficient of imaginary potential exceeds this critical value, the power of the beam increases and results in the unstable beam propagation. The stability of the stationary solution against small perturbation has been studied using linear stability analysis. The imaginary eigen value is zero when the coefficient of the imaginary potential is low. Above the phase transition point, the imaginary eigen value becomes comparable with the real eigen value and hence the solution becomes linearly unstable.

1. Introduction

The study of dissipative systems with non Hermitian Hamiltonians have attracted a lot of attention in recent years. Such non Hermitian operators posses real spectra, provided that they obey parity-time ($\mathcal{PT}$) symmetry. This concept was introduced by Bender and Boettcher in 1998 [1–3]. According to their view, those non-Hermitian Hamiltonian share common set of eigen functions with the $\mathcal{PT}$ operator posses real spectra. The $\mathcal{PT}$ operations can be stated as follows [4–7]. Under the action of parity operator, $\hat{p} \to -\hat{p}, \hat{x} \to -\hat{x}$ ($\hat{p}$ and $\hat{x}$ are the momentum and position operators respectively) and the action of time reversal operator results $\hat{p} \to -\hat{p}, \hat{x} \to \hat{x}, i \to -i$. The Hamiltonian of a physical system is $\hat{H} = (\hat{p}^2/2m) + V(x)$, where m is the mass and V is the complex potential. Under time reversal operation, $\hat{T}\hat{H} = (\hat{p}^2/2m) + V^*(x)$. After the parity operation $\hat{P}\hat{T}\hat{H} = (\hat{p}^2/2m) + V^*(-x)$ and $\hat{H}\hat{P}\hat{T} = (\hat{p}^2/2m) + V(x)$. For the Hamiltonian to be $\mathcal{PT}$ symmetric, $\hat{P}\hat{T}\hat{H} = \hat{H}\hat{P}\hat{T}$. That is possible only if the condition $V(x) = V^*(-x)$ is satisfied [2]. In order to satisfy the above condition, the real part of the complex potential must be an even (symmetric) function of position, where as the imaginary part should be an odd (antisymmetric) function. The dissipative systems, which include exactly balanced linear gain and loss, are described by non-Hermitian Hamiltonians, whose Hermitian and non-Hermitian parts are spatially even and odd, respectively. It was also demonstrated that when the non-Hermitian Hamiltonians are $\mathcal{PT}$ symmetric, they undergo a phase transition ($\mathcal{PT}$ symmetry breaking) above a critical threshold, above which the eigen value spectrum becomes partially complex [8]. This concept of $\mathcal{PT}$

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symmetry of a non-Hermitian Hamiltonian generalizes quantum mechanics in a complex domain [5,9]. $\mathcal{PT}$ symmetry finds applications in various areas of physics ranging from $\mathcal{PT}$ symmetric quantum oscillators to linear and nonlinear optics [1,4].

$\mathcal{PT}$ symmetric structures have been realized in optics by Christodoulides et al. [6,10]. Such systems can be realized through the inclusion of gain/loss regions in guided wave geometries. In the optical systems the complex refractive index distribution, $n(x) = n_r(x) + in_i(x)$ plays the role of the optical potential [10–13]. The $\mathcal{PT}$ symmetry condition implies that the index wave guiding profile $n_r(x)$ is an even function in the transverse direction while the loss or gain term $n_i(x)$ is an odd function. $\mathcal{PT}$ symmetric optical systems have been experimentally demonstrated in AlGaAs [14], photorefractive materials [6], silicon, fiber optics, and light-written guides in glass [15–17], etc. The observation of $\mathcal{PT}$ symmetry in linear optical systems led to its generalization to the nonlinear case, which provides several interesting predictions. It was also demonstrated that $\mathcal{PT}$ symmetric nonlinear optical systems can support soliton solutions [12,18]. Recently, soliton beam propagation in $\mathcal{PT}$ symmetric optical media has been a subject of intense research because the beam dynamics in such systems can exhibit unique characteristics such as double refraction, power oscillations, nonreciprocal diffraction patterns, etc. [19,20]. The stability of the optical solitons in nonlinear $\mathcal{PT}$ symmetric systems is investigated in a number of complex potentials like $\mathcal{PT}$ symmetric periodic potential, hyperbolic Scarf [21–23] potential, etc. Studies of $\mathcal{PT}$ symmetry have also been extended from nonlinear lattice to double channel coupled wave guides [24–26], double-well potentials and asymmetric optical amplifier [27]. Spatial solitons in self focusing and defocusing Kerr media and nonlocal media with $\mathcal{PT}$ symmetric potentials have also been investigated [28,30].

This work analyzes the soliton beam propagation in a $\mathcal{PT}$ symmetric system, which is characterized by the nonlinear Schrödinger equation with a complex potential with competing gain and loss profile. The paper is organized as follows. In Section 2, the stationary solutions of the system have been studied in different imaginary potentials. In Section 3, the beam dynamics has been analyzed using variational and numerical methods. In Section 4, the stability of the stationary solution against small perturbation has been analyzed using linear stability analysis. Section 5 concludes the paper.

2. Stationary solution

The beam evolution in a (2 + 1)D self focusing Kerr media is governed by the normalized nonlinear Schrödinger equation [29–31].

$$i\psi_z + \psi_{xx} + \psi_{yy} + V\psi + \beta|\psi|^2\psi = 0. \tag{1}$$

The suffixes z, x and y stand for the partial derivatives with respect to z, x and y respectively, V is the complex potential and $\beta = +1$ corresponds to the self focusing nonlinearity. For radial symmetry, $x^2 + y^2 = r^2$ and $\nabla^2 = (1/r)(\partial/\partial r) + (\partial^2/\partial r^2)$. Then Eq. (1) becomes

$$i\psi_z + \psi_{rr} + \frac{1}{r}\psi_r + V\psi + \beta|\psi|^2\psi = 0. \tag{2}$$

A $\mathcal{PT}$ symmetric potential can be implemented through the judicious inclusion of lumped amplification, $V_r(r)$, which is associated with index guiding and a loss/gain distribution term, $V_i(r)$. Then the beam dynamics in a $\mathcal{PT}$ symmetric graded index Kerr media is governed by

$$i\psi_z + \psi_{rr} + \frac{1}{r}\psi_r + (V_r(r) + iV_i(r))\psi + \beta|\psi|^2\psi = 0. \tag{3}$$

$\mathcal{PT}$ symmetry demands that $V_r(-r) = V_r(r)$ and $V_i(-r) = -V_i(r)$. The complex $\mathcal{PT}$ symmetric potential is chosen in the form $V(r) = (V_r/2)r^2 + iV_ir$, where V_r and V_i are the coefficients of real and imaginary potential.

Stationary solution of Eq. (3) can be of the form $\psi(r, z) = \phi(r)e^{-i\mu z}$ [28], where $\phi(r)$ is the nonlinear eigen mode which is a complex function of r and μ is the corresponding propagation constant. The nonlinear eigen value equation satisfied by $\phi(r)$ is given by

$$\phi_{rr} + \frac{1}{r}\phi_r + (V_rr^2 + iV_ir)\phi + \beta|\phi|^2\phi = -\mu\phi. \tag{4}$$

The differential equation (4) has been studied to analyze the beam intensity in different imaginary potentials. The modulus, real and imaginary components of the stationary solution are shown in Fig. 1. Figure shows that the imaginary part of the solution starts to evolve when the coefficient of imaginary potential exceeds a critical threshold, which is referred as the phase transition or $\mathcal{PT}$ symmetry breaking. At $\mu = 0.5$, the imaginary component of the solution is negligible when $V_i < 0.6$ (as shown in the upper row of figure, Fig. 1(a)–(c)). When $V_i \geq 0.6$, the solutions possess real and imaginary components. Similarly, at $\mu = 1.5$, the solution is complex when $V_i \geq 1.2$ (lower row of Fig. 1). The phase transition point increases with the propagation constant. The variation of V_i with μ at constant $V_r(V_r = 1)$ is shown in Fig. 2.

3. Beam dynamics in $\mathcal{PT}$ symmetric potential

The semi-analytical results of Eq. (3) are given by the variational analysis [25,28–36] for the solution of the form,

$$\psi(r, z) = \phi(r, z)\exp(-i\mu z), \tag{5}$$