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Sliding mode control for synchronization of fractional permanent magnet synchronous motors with finite time

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ABSTRACT

Chaos synchronization of motor system has been central to recent experimental and theoretical investigations. This paper concentrates on the fractional version of permanent magnet synchronous motors (PMSM). The necessary condition for the emergence of chaos in fractional PMSM is deduced. Based on the fractional stability theory, a sliding mode control scheme for synchronization of fractional PMSM is developed. This scheme is simple with a single control input and can realize the asymptotical synchronization in a finite time. Finally, simulation results are provided to verify the effectiveness of the proposed scheme.

1. Introduction

Although holds a history of more than 300 years [1], the application of fractional calculus to different fields is just a recent topic of interests [2,3]. In comparison to integer calculus, fractional calculus has been proven to be a more suitable mathematical tool to model the reality since it can offer a deeper insight into the physical processes underlying a long range memory behavior [4].

The PMSM has superior features such as low manufacturing cost, simple structure, more torque per weight and efficiency, and naturally has been widely used in direct-drive robotic applications, especially in industrial applications for low–medium power range [5–7]. Investigations show that when system parameters fall into certain area, PMSM exhibits chaotic behavior, which will decrease the system performance, and is highly undesirable in most engineering applications. Many efficient control strategies are proposed to eliminate the chaotic behavior in PMSM.

In addition, synchronization of chaotic motor, which implies the slave motor is controlled to work following the master motor in the same rhyme concerning the amplitude and the angular phase, has been attracted to recent focus of investigations. For instance, based on the variable substitution control strategy, Liu realized chaos synchronization of Brushless DC motors [8]. Ge investigated the complete, lag and anticipated synchronization of BLDCM chaotic system [9,10]. Based on the theory of passive control, Su considered the synchronization problem of chaotic PMSM via output feedback [11]. Verrelli realized the synchronization problem of PMSM based on Fourier approximation theory [12]. All these existing synchronization schemes for chaotic motors concentrate on the integer version rather than the fractional one. So, it is significant to consider the chaos synchronization of fractional motor system.

In this paper, we mainly concentrate on the fractional PMSM. The necessary condition for the emergence of chaos in fractional PMSM is deduced. And based on the fractional stability theory, a sliding mode control scheme for synchronization of fractional PMSM is developed. This scheme is simple with a single control input and can realize the asymptotical synchronization in a finite time. Finally, simulation results are provided to verify the effectiveness of the proposed scheme.

2. Preliminaries for fractional theory

The definition of fractional derivative introduced by Caputo is a common notation, which is described as bellow [3].

Definition 1. For function $f(t)$ with respect to t, the α-order fractional derivative is defined by

$$D^{\alpha}f(t)=\frac{1}{\Gamma(n-\alpha)}\int_{t_0}^{t}(t-\tau)^{-\alpha+n-1}f^{(n)}(\tau)\mathrm{d}\tau \tag{1}$$

where $\Gamma(\cdot)$ is the Gamma function, n is an integer and satisfies $n=[\alpha]+1$, $[\alpha]$ is the integer part of α.

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A general property of the fractional derivative is recalled here, which will be used.

Property 1. The Caputo fractional derivative operator satisfies the additive index law

$$D^{\alpha_1}D^{\alpha_2}x(t)=D^{\alpha_2}D^{\alpha_1}x(t)=D^{\alpha_1+\alpha_2}x(t)$$

For the stability analysis of fractional-order system, we have the following theorems.

Theorem 1 ([13]). Consider the fractional dynamical system described as

$$D^{\alpha}X=F(X) \tag{2}$$

with $X\in R^n$, $F(X)\in R^n$ and $\alpha\in(0,1]$. The necessary condition for the emergency of chaos in a commensurate fractional system is $\alpha\pi/2-\min\left\{\left|arg(\lambda_i)\right|\right\}\ge 0$, where λ_i, $i=1, 2, \ldots, n$ is the characteristic root of the Jacobian matrix $J=\partial F(X)/\partial X$.

Theorem 2 ([14]). Consider the fractional system (2), if there exists a matrix $Q=Q^T>0$, such that the following condition can be catered

$$\sum = X^T Q D^{\alpha}X\le 0 \tag{3}$$

then system (2) is asymptotically stable.

Theorem 3 ([2]). Suppose the equilibrium point of system (2) be $X=0$. If there exists a continuously differentiable function $V(X, t)$: $R^n\times[0,\infty)\to R$, such that

$$s\gamma_1\left\|X\right\|\le V(X,t)\le\gamma_2\left\|X\right\| \tag{4}$$

$$\dot{V}(X,t)\le-\eta V(X,t) \tag{5}$$

where $\gamma_1, \gamma_2, \eta>0$. Then system (2) is Miattag–Leffler (asymptotic) stable.

3. The fractional PMSM

3.1. Model of PMSM

The dimensionless mathematical model of PMSM is depicted as [15]

$$\begin{cases}\dfrac{di_d}{dt}=-i_d+\omega i_q+u_d\\ \dfrac{di_q}{dt}=-i_q-\omega i_d+\gamma\omega+u_q\\ \dfrac{d\omega}{dt}=\sigma\left(i_q-\omega\right)-T_L\end{cases} \tag{6}$$

in which i_d, i_q denote the stator currents; ω is the rotor angular frequency; the stator voltages $u_d=K_TLU_d/bR^2\ +K_T{}^2/bR+\gamma$, $u_q=K_TLU_q/bR^2$, where $L=L_d=L_q$; T_L is the external load torque; $\sigma=bL/JR$ and γ are the operating parameters. The investigation shows that all the equilibria become unstable with the system parameters σ and γ falling into a certain area, thereby the PMSM exhibits chaos. In this paper we only take the case that the motor is running freely without loading, and the external inputs are set to zero, i.e., $u_d=u_q=T_L=0$.

3.2. The fractional PMSM

Now, we consider the fractional-order PMSM system with $\alpha\in(0, 1]$:

$$\begin{cases}\dfrac{d^{\alpha}i_d}{dt^{\alpha}}=-i_d+\omega i_q\\ \dfrac{d^{\alpha}i_q}{dt^{\alpha}}=-i_q-\omega i_d+\gamma\omega\\ \dfrac{d^{\alpha}\omega}{dt^{\alpha}}=\sigma\left(i_q-\omega\right)\end{cases} \tag{7}$$

By calculating, we can easily get three equilibrium points of system (7):

$$P_0=(0,0,0),\quad P_{1,2}=\left(\gamma-1,\pm\sqrt{\gamma-1},\pm\sqrt{\gamma-1}\right) \tag{8}$$

and the corresponding Jacobian matrix:

$$J=\begin{bmatrix}-1 & \omega_* & i_{q_*}\\ -\omega_* & -1 & \gamma-i_{d_*}\\ 0 & \sigma & -\sigma\end{bmatrix} \tag{9}$$

where $\left(i_{d_*}, i_{q_*}, \omega_*\right)$ denote the equilibrium point of system (7).

The three equilibrium points of system (7) are $P_0=(0, 0, 0)$, $P_1=(49, 7, 7)$, $P_2=(49, -7, -7)$ with $\sigma=4$, $\gamma=50$. Accordingly the corresponding eigenvalues are

$$P_0:\lambda_1=-16.7215,\lambda_2=11.7215,\lambda_3=-1$$

$$P_{2,3}:\lambda_1=0.3443+7.6477\text{i},\lambda_2=0.3443-7.6477\text{i},\lambda_3=-6.6887$$

Therefore, with the aid of Theorem 1, $\alpha>0.9714$ is the necessary condition for the existence of chaos in the fractional PMSM system when $\sigma=4$, $\gamma=50$, which is verified in Fig. 1 with $\alpha=0.98$.

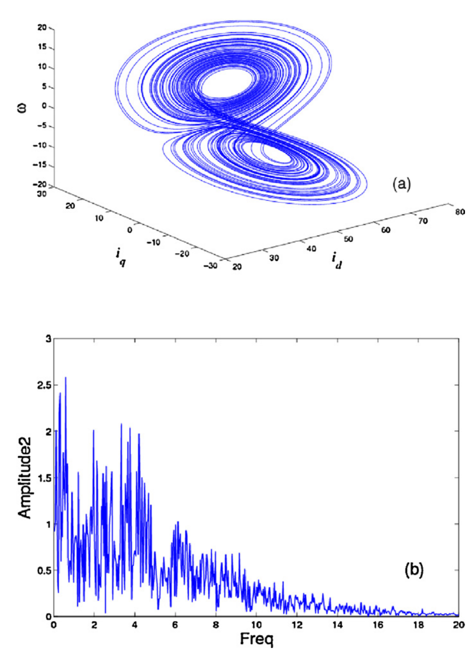

Fig. 1. Chaotic behavior of fractional PMSM: (a) chaotic attractor; (b) power spectral density.