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Stopping criterion for anisotropic image diffusion

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ABSTRACT

Coherence-enhancing diffusion filtering is a striking application of the anisotropic diffusion in image processing. The technique deals with the problem of completion of interrupted lines and enhancement of flow-like features in fingerprint images. However, an anisotropic diffusion process is an iterated process, initializes with a poor quality image, and converges at the end towards a completely blurred image, with no structure surviving at the end. In anisotropic diffusion, one important question is how to find boundary between the under-smoothing and over-smoothing regions of the anisotropic diffusion process. The entropy change is found to be one such measure to describe that boundary adequately and thus provides a reasonable stopping rule for anisotropic diffusion. Numerical experiments with test pattern images confirm the desirable qualities of gap-closing and flow-enhancing qualities, along with the identification of frontier of useful smoothing. The proposed scheme is evaluated with the help of simulated images, and compared with other state of the art schemes using an objective criterion.

1. Introduction

Image smoothing and denoising is one of most fundamental and important image processing techniques. The basic principle of image denoising is to filter the noise by some kind of filter, and keep the original image content (especially fine structures, such edges and lines) as intact as possible [1,2]. In many image processing problems, we often come across the enhancement of elongated structures, such as ridges, edges, and oriented texture patterns in noisy images [3–6]. One classic example that can be cited is the case of enhancing noisy image data [7–9]. Many methods, proposed in the literature, are based on implementing a non-linear anisotropic diffusion equation on noisy images. The idea was pioneered by Nitzbeg and Shiota [10] and Cottet et al. [11]. Later on, Weickert [12] put forward a formal method for enhancing elongated structure, referred to as Coherence Enhanced Diffusion (CED). The CED works by steering the diffusion process in a particular direction with the help of a diffusion tensor. The design was further generalized by adopting a diffusion matrix to learn the local structure iteratively [13].

The basic idea of CED is to smooth a degraded image by applying a repeated nonlinear diffusion process. The diffusion tensor of nonlinear diffusion process allows anisotropic smoothing by acting mainly along the preferred structure direction. This so-called coherence orientation is determined by the eigenvector of the structure tensor with the smallest eigenvalue. Anisotropic diffusion is normally implemented using an approximation of the generalized diffusion equation. The new image in the family is computed by applying this equation to the previous image. Consequently, anisotropic diffusion becomes an iterative process where a relatively simple set of computation is used to compute each successive image in the family. This process is continued until a sufficient degree of smoothing is obtained. The CED process initializes with a noisy image and converges to a constant image. Thus, we have an under-smooth situation that ultimately turns into an over-smooth one.

Overestimating the stopping time will result in an over-smoothed blurry image while under-estimating it may leave the noise in the image unfiltered. Therefore, it is crucial that an appropriate time is selected in an automatic way. The activity in literature can be divided into two broad categories. One deals with an additive noise model. Treating the noisy image as the result of noise addition, they try to stop the diffusion where the correlation between the diffused image and the initial noisy image minimizes [12]. The authors in [14] introduced a multigrid algorithm using a normalized cumulative periodogram. A frequency approach to the problem was presented in [15]. Whereas, [16] uses the extent of noise smoothing in every iteration as a stopping parameter for diffusion. Later on, a spatially varying stopping method was introduced that increased the computational cost significantly [17]. By identifying it as a

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Lyapunov functional of a large class of scalar-valued nonlinear diffusion filters, Weigert [18] introduced decreasing the variance of an image as a stopping tool. The second category deals with examining the entropy of the diffused image, and a stopping criterion was developed to deal with its distance from the entropy of the original noisy image [12]. Our research work also introduces a new stopping rule based on changes in entropy of the diffused image as it evolves. Since the change in entropy is related to information loss that results when an image structure is disturbed, we propose that a maximum entropy change may be a good stopping time for the diffusion process.

The rest of this paper is organized as follows. In Section 2, coherence enhanced diffusion is discussed. The discrete image as a spatial distribution is described in Section 3. Section 4 is about the entropy of coherence enhanced diffusion followed by Results and discussion in Section 5. Finally the paper is concluded in Section 6.

2. Coherence enhanced diffusion

Consider an input image $L(x, y)$. The anisotropic scale-space for the image $L(x, y)$ can be constructed by realizing the heat equation, given by:

$$\partial_t L = \nabla(D\nabla L), \tag{1}$$

where D is the 2×2 diffusion matrix, adapted to the local image structure, via a structural descriptor, called the second-moment matrix μ, defined as:

$$S = \begin{pmatrix} s_{11} & s_{12} \\ s_{12} & s_{22} \end{pmatrix} = \begin{pmatrix} L_{x,\sigma}^2 & L_{x,\sigma}L_{y,\sigma} \\ L_{x,\sigma}L_{y,\sigma} & L_{y,\sigma}^2 \end{pmatrix}, \tag{2}$$

where L_x^2, L_xL_y and L_y^2 represents the gaussian derivative filters, in the x and y directions. This symmetric 2×2 matrix has two dominant eigenvalues μ_1 and μ_2, given by:

$$\mu_1 = \frac{1}{2}(s_{11} + s_{12} + \alpha), \qquad \mu_2 = \frac{1}{2}(s_{11} + s_{12} - \alpha), \tag{3}$$

where

$$\alpha = \sqrt{(s_{11} - s_{22})^2 + 4s12^2}. \tag{4}$$

Since the eigenvalues integrate the variation of the gray values within a neighborhood, they describe the average contrast in the eigen directions v and w. With the help of the eigenvalues of the structure matrix a useful information can be obtained on the coherence. The expression $(\mu_1 - \mu_2)^2$ is large for anisotropic structures and tends to zero for isotropic structures, while constant areas are characterized by $\mu_1 = \mu_2 = 0$, straight edges by $\mu_1 \gg \mu_2$ or $\mu_2 \gg \mu_1$, corners by $\mu_1 = \mu_2 \gg 0$, and the flat region by $\mu_1 = \mu_2 \approx 0$.

These eigenvalues are associated with their respective eigenvectors. The first normalized eigenvector can be written as $(\cos\theta, \sin\theta)^T$, and the second orthogonal eigenvector comes out as $(-\sin\theta, \cos\theta)^T$. The parameter θ represents the orientation field of the given image. The orientation of the eigenvector corresponding to the smaller eigenvalue μ_2 is called coherence orientation. This orientation has the lowest fluctuations. The eigenvalues of the diffusion matrix are assembled as

$$\lambda_1 = c_1$$

$$\lambda_2 = \begin{cases} c_1 & \text{if } \mu_1 = \mu_2; \\ c_1 + (1 - c_1)\exp\left(\dfrac{c_2}{(\mu_1 - \mu_2)^2}\right) & \text{else,} \end{cases} \tag{5}$$

where $0 < c_1 \ll 1$ and $c_2 > 0$.

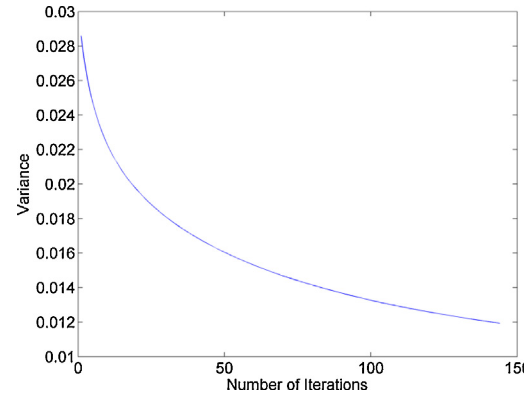

Fig. 1. This graph shows the monotonic decreasing behavior of CED.

The diffusion matrix D can be reconstructed with help of its eigenvalues and eigenvectors as:

$$d11 = \lambda_1\cos^2\theta + \lambda_2\sin^2\theta$$
$$d12 = (\lambda_1 - \lambda_2)\sin\theta\cos\theta \tag{6}$$
$$d22 = \lambda_1\sin^2\theta + \lambda_2\cos^2\theta$$

Once the diffusion matrix is constructed, the evaluation process is set to start. The diffusion process proceeds in four steps.

1. Calculate the second-moment matrix for each pixel.
2. Construct the diffusion matrix for each pixel.
3. Calculate the change in intensity for each pixel as $\nabla(D\,\nabla\,L)$.
4. Update the image using the diffusion equation as:

$$L^{t+\Delta t} = L^t + \Delta t \times \nabla(D\nabla L) \tag{7}$$

The above four steps are iterated under a given stopping rule.

The question that always arises in the diffusion process is where to stop. The stopping rule for the diffusion process can be defined in terms of measures that reveal monotonic behavior as we move deeper into the evaluation process. By identifying it as a Lyapunov functional of a large class of scalar-valued nonlinear diffusion filters, Weickert [18] has shown that the variance $\eta^2(L^t)$ is indeed one such measure that is monotonously decreasing. This monotonic decreasing behavior is also evident in the graph depicted in Fig. 1. What can be seen from the graph is that it is fast decreasing at the beginning, but towards the end, it becomes saturated, providing convergence. Thus, by bounding the relative change at the variance one can define the diffusion stopping rule. However, this rule does not guarantee an optimal time to stop the process. It is based on the user defined ratio of diffused image variance to that of initial image variance. This ratio might be useful if we want to compare various different diffusion schemas, but its utility, to provide a well-diffused image with all the important structure cleaned but in-tact, may be limited.

3. A discrete image as spatial distribution

Consider a discrete image $L(x, y)$, where x is the row index and y is the column index. This discrete image can be realized as a spatially distributed light intensity [19]. Each spatial location, that is (x, y) in the image, registers the number of light quantum-hit. In this way we may define

$$p(x, y) = \frac{L(x, y)}{\sum_x \sum_y L(x, y)} \tag{8}$$