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Complex hybrid synchronization of complex-variable dynamical network via impulsive control

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ABSTRACT

In this paper, complex hybrid synchronization of network coupled with complex-variable chaotic systems is investigated. The complex-variable dynamical network is synchronized onto a given orbit with respect to a complex matrix using impulsive control. Adaptive strategy is adopted to design adaptive impulsive controllers, which is universal for different dynamical networks. Further, it can relax the restriction on impulsive intervals. Based on impulsive stability theory and Lyapunov function method, several synchronization criteria for achieving complex hybrid synchronization are derived. Numerical examples are provided to show the effectiveness of the theoretical results.

1. Introduction

In recent years, many kinds of projective synchronization with respect to constant matrices are introduced for better explaining the complex phenomenon in complex network coupled with dynamical systems. Complete synchronization [1–7] and anti-synchronization [7,8] are two special projective synchronization manners with respect to identity and negative identity matrices. Projective synchronization is obtained if the matrix is chosen as scalar matrix [9–12]. Specially, hybrid projective synchronization is achieved if the matrix is chosen as general diagonal matrix [13–15]. In [14], adaptive hybrid projective synchronization of two different chaotic systems with respect to a diagonal matrix is studied.

Recently, many complex-variable chaotic and hyperchaotic systems are introduced for better describing the physical systems processes and the synchronization of coupled complex-variable chaotic systems are well investigated [16–22]. In Refs. [16–18], complex-variable Lorenz system is introduced and used to describe and simulate rotating fluids and detuned laser. In [19], the complex-variable Chen and Lü systems are introduced and the global synchronization are studied via active control. In [20], the hybrid projective synchronization in chaotic complex dynamical system with respect to a scaling matrix is studied. In [22], the synchronization of dynamical network coupled with complex-variable chaotic systems is studied.

All the projective factors in the above synchronization are real number or real matrix, which means that the drive-response systems evolve in the same or inverse direction simultaneously. However, for complex-variable dynamical systems, the drive-response systems may evolve in different directions with a constant intersection angle in complex space. That is to say, the projective factors can be complex numbers or matrices. In [23], the complex projective synchronization of coupled complex-variable chaotic systems with respect to complex number is investigated. In [24], the complex hybrid synchronization in drive-response complex-variable chaotic systems with respect to complex diagonal matrix is considered. On the other hand, impulsive control method has been widely used to design proper controllers for achieving synchronization of coupled dynamical systems [25–31].

Motivated by the above discussions, in this paper, the complex hybrid synchronization of dynamical network coupled with complex-variable chaotic systems is investigated via impulsive control. Noticeably, adaptive strategy is adopted to design adaptive impulsive controllers, which is universal for different dynamical networks and can relax the restriction on impulsive intervals. According to impulsive stability theory and Lyapunov function method, several sufficient conditions for achieving complex hybrid synchronization are provided and verified by numerical examples.

The rest of this paper is organized as follows. Section 2 introduces the model and some preliminaries. Section 3 studies the complex hybrid synchronization of complex-variable dynamical network through designing proper impulsive controllers. Section 4 provides several numerical simulations to verify the effectiveness of the theoretical results. Section 5 concludes this paper.

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Notation Throughout this paper, for symmetric matrix M, the notation $M>0$ ($M<0$) means that the matrix M is positive definite (negative definite). For any complex number (or complex vector) x, the notations x^r and x^i denote its real and imaginary parts respectively and $\bar{x}$ denotes the complex conjugate of x.

2. Model and preliminaries

Consider a network consisting of N individuals indexed by $k=1, 2, \ldots, N$, described by a complex-variable chaotic system

$$\begin{aligned} \dot{x}_k(t) &= f(x_k(t), z_k(t)), \\ \dot{z}_k(t) &= g(x_k(t), z_k(t)), \end{aligned} \tag{1}$$

where $x_k(t)=(x_{k1}(t), x_{k2}(t), \ldots, x_{km}(t))^T \in C^m$ is an m-dimension complex state vector with $x_{kl}(t)=x_{kl}^r(t)+jx_{kl}^i(t)$, $l=1, 2, \ldots, m$ and $j=\sqrt{-1}$, $z_k(t)=(z_{k1}(t), z_{k2}(t), \ldots, z_{kn}(t))^T \in R^n$ is an n-dimension real state vector. $f: C^m \times R^n \to C^m$ is a complex-valued vector function and $g: C^m \times R^n \to R^n$ is a real-valued vector function.

Let $X_k(t)=(x_k^T(t), z_k^T(t))^T$ be the state variable of the kth node, and $F(X_k(t))=(f^T(x_k, z_k), g^T(x_k, z_k))^T$ be the node dynamics. The complex-variable dynamical network can be described by

$$\dot{X}_k(t) = F(X_k(t)) + \sum_{l=1}^{N} a_{kl}\Gamma X_l(t), \tag{2}$$

where $k=1, 2, \ldots, N$, $\Gamma=\mathrm{diag}(\gamma_1, \gamma_2, \ldots, \gamma_{m+n}) \in R^{(m+n)\times(m+n)}$ is inner coupling matrix. $A=(a_{kl}) \in R^{N\times N}$ is the zero-row-sum outer coupling matrix, which determines the topology of the network, defined as: if node k is affected by node l ($l \neq k$), then $a_{kl} \neq 0$; otherwise, $a_{kl}=0$.

Let $s(t)$ be a solution of an isolated node satisfying $\dot{s}(t)=F(s(t))$, and $H=\mathrm{diag}(h_1, h_2, \ldots, h_{m+n})$ with $h_\rho=e^{j\theta}$ for $1\le\rho\le m$ and $h_\rho=1$ for $m+1\le\rho\le m+n$, where $\theta\in[0, 2\pi)$.

Definition 1. Network (2) is said to achieve complex hybrid synchronization with respect to complex matrix H, if

$$\lim_{t\to\infty}\|X_k(t)-Hs(t)\|=0, \quad k=1, 2, \ldots, N.$$

The objective here is to achieve the complex hybrid synchronization of network (2) through designing proper impulsive controller. Then, the network (2) with impulsive controllers can be written as

$$\begin{aligned} \dot{X}_k(t) &= F(X_k(t)) + \sum_{l=1}^{N} a_{kl}\Gamma X_l(t), \quad t\neq t_\sigma, \\ X_k(t_\sigma^+) &= X_k(t_\sigma^-) + b_\sigma(X_k(t)-Hs(t)), \quad t=t_\sigma, \end{aligned} \tag{3}$$

where $\sigma=1, 2, \ldots$, $\{t_\sigma\}$ is a discrete constant set satisfying $0=t_0<t_1<\cdots<t_\sigma<\cdots$, $t_\sigma\to\infty$ as $\sigma\to\infty$, $X_k(t_\sigma^+)=\lim_{t\to t_\sigma^+}X_k(t)$, $X_k(t_\sigma^-)=\lim_{t\to t_\sigma^-}X_k(t)$. $b_\sigma\in(-2, 0)$ is impulsive gain at $t=t_\sigma$, and $b_\sigma=0$ for $t\neq t_\sigma$. Any solutions of (3) are assumed to be left continuous at each $t=t_\sigma$, i.e., $X_k(t_\sigma)=X_k(t_\sigma^-)$.

Let $e_k(t)=X_k(t)-Hs(t)$ be the synchronization errors, then one has the following error system

$$\begin{aligned} \dot{e}_k(t) &= F(X_k(t)) - HF(s(t)) + \sum_{l=1}^{N} a_{kl}\Gamma e_l(t), \quad t\neq t_\sigma, \\ e_k(t_\sigma^+) &= (1+b_\sigma)e_k(t), \quad t=t_\sigma. \end{aligned} \tag{4}$$

For achieving the complex hybrid synchronization, the following assumption and lemma are needed.

Assumption 1. Suppose that there exists a positive constant L such that the vector function $F(X_k(t))$ satisfies

$$\begin{aligned} &(X_k(t)-Hs(t))^T(\overline{F(X_k(t))-HF(s(t))}) + (F(X_k(t)) \\ &\quad -HF(s(t)))^T(\overline{X_k(t)-Hs(t)}) \le L(X_k(t)-Hs(t))^T(\overline{X_k(t)-Hs(t)}), \end{aligned} \tag{5}$$

where $k=1, 2, \ldots, N$.

Lemma 1. [23] *For any two complex numbers α and β, and any real constant $\eta>0$, the following inequality holds:*

$$\alpha\bar{\beta}+\bar{\alpha}\beta \le \eta\alpha\bar{\alpha}+\eta^{-1}\beta\bar{\beta}.$$

3. Complex hybrid synchronization

In this section, some sufficient conditions for achieving the complex hybrid synchronization of complex-variable dynamical network (3) are derived.

In what follows, let $e(t)=(e_1^T(t), e_2^T(t), \ldots, e_N^T(t))^T$, $\tau_\sigma=t_\sigma-t_{\sigma-1}$ be the impulsive intervals, $\beta_\sigma=(1+b_\sigma)^2$ for $\sigma=1, 2, \ldots$, and λ_1 be the largest eigenvalue of $(A+A^T)\otimes\Gamma$.

Theorem 1. *Suppose that Assumption 1 holds. If there exists a constant $\alpha>0$ such that the following conditions*

$$\ln\beta_\sigma+\alpha+(L+\lambda_1)\tau_\sigma<0, \quad \sigma=1, 2, \ldots, \tag{6}$$

hold, then the complex hybrid synchronization of network (3) can be achieved.

Proof. Consider the following Lyapunov function

$$V(t)=\sum_{k=1}^{N} e_k^T(t)\bar{e_k}(t).$$

When $t\neq t_\sigma$, the derivative of $V(t)$ with respect to t along the trajectories of (4) is

$$\begin{aligned} \dot{V}(t) &= \sum_{k=1}^{N}(e_k^T(t)\bar{\dot{e}_k}(t)+\dot{e}_k^T(t)\bar{e_k}(t)) \\ &= \sum_{k=1}^{N}(e_k^T(t)(\overline{F(X_k(t))-HF(s(t))})+(F(X_k(t))-HF(s(t)))^T\bar{e_k}(t)) \\ &\quad + \sum_{k=1}^{N}\sum_{l=1}^{N} a_{kl}(e_k^T(t)\Gamma\bar{e_l}(t)+e_l^T(t)\Gamma\bar{e_k}(t)). \end{aligned}$$

Then, according to Assumption 1, one has

$$\dot{V}(t)\le Le^T(t)\bar{e}(t)+e^T(t)((A+A^T)\otimes\Gamma)\bar{e}(t)\le(L+\lambda_1)e^T(t)\bar{e}(t),$$

which gives

$$V(t)\le e^{(L+\lambda_1)(t-t_{\sigma-1})}V(t_{\sigma-1}), \quad t\in(t_{\sigma-1}, t_\sigma). \tag{7}$$

When $t=t_\sigma$, one has

$$V(t_\sigma^+)=\sum_{k=1}^{N}e_k^T(t_\sigma^+)\bar{e_k}(t_\sigma^+)=(1+b_\sigma)^2\sum_{k=1}^{N}e_k^T(t_\sigma^-)\bar{e_k}(t_\sigma^-)=\beta_\sigma V(t_\sigma^-). \tag{8}$$

Combining inequalities (7) and (8), for positive integer σ, the following inequality can be proved according to mathematical induction

$$V(t_\sigma^+)\le V(t_0^+)\prod_{\kappa=1}^{\sigma}\beta_\kappa e^{(L+\lambda_1)\tau_\kappa}.$$

From conditions (6), one has

$$\beta_\sigma e^{(L+\lambda_1)\tau_\sigma}<e^{-\alpha}, \quad \sigma=1, 2, \ldots$$