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Influence of detuning on entanglement in non-Markovian channels

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ABSTRACT

Quantum entanglement between two qubits was obtained by solving the model of two qubits coupled to two independent reservoirs. This paper studied the influences of dissipative environment, detuning between transition frequency of the qubit and center frequency of a cavity. Results showed that: (1) In the non-Markovian channel, a certain amount of detuning was conducive to maintain and improve the entanglement of a system. (2) On the other hand, in a two-sided amplitude damping channel, the composite effect of two independent dissipative channels would appear through the entanglement between two qubits. Given the composite effect of two independent channels, the entanglement dynamics not only affected the Markovian process but also hindered the backflow effect of the non-Markovian channel, which was not conducive to maintain the entanglement between qubits.

1. Introduction

Quantum entanglement is the key resource of quantum information and quantum computation. It is not only a powerful tool to test the basic problems of quantum mechanics, but also a physical foundation for building future quantum information science [1–3]. Quantum properties, however, are very fragile. In the real physical world, the interaction between a quantum system and its surrounding environment may lead to irreversible loss of information. In order to ensure the realization of correct computing of quantum logic and quantum computation, people have undertaken many studies on entanglement sudden death (ESD), including not only how to avoid ESD but also how to restore entanglement in the two-qubit system [4–6].

The interaction between the quantum system and environment can be divided into Markovian and non-Markovian processes. For the Markovian process, information transferred from the quantum system to the environment will not return to the system from the environment, thereby preventing the system from evolving to the next level. Considering the memory effects in the non-Markovian process, the information previously transferred from the quantum system to environment will feedback to system after some time, thereby influencing the next evolution of the system. Thus, in the non-Markovian process, the dynamic behavior of the system will be more complex [7–9]. One found that entanglement evolution of semiconductor quantum dots and superconducting qubit are typical non-Markovian processes [10].

Decoherence is one of the most important problems in quantum information processing. The description of this difficult problem usually involves various approximations. During the dynamic evolution, the system and the bath are mixed, and a perturbative treatment is required so that we can trace out the degrees of freedom of the bath. This perturbation is known as the Born approximation. Moreover, if the time scale of the bath is much shorter than that of the system, the Markovian approximation is often applied [11]. Recent studies have shown that non-Markovian quantum processes play an increasingly important role in many fields of physics. However, neither the Born approximation nor the Markovian approximation is suitable for the study of quantum properties in non-Markovian process.

An effective factorization method that propitious to non-Markovian process was developed by Konrad et al., who provided a direct relationship between the initial and final entanglements between two qubits with one qubit subject to incoherent dynamics. The factorization law showed that given any one-sided quantum channel, the concurrence of the output state corresponding to any initial pure input state of interest could always be equivalently obtained by the product of the concurrence of the input state and

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that of the output state with the maximally entangled state as an input state [12,13].

Although the entanglements of open system have been studied extensively, many problems still need exploring. This paper comprehensively considered the influences of detuning between the central frequency of the cavity and the transition frequency of qubit on the entanglement between two qubits in non-Markovian processes. In addition, we compared the entanglement behavior of two qubits in a two (or single)-sided amplitude damping channel in the same environment spectrum density, thereby yielding more persuasive results.

2. Model

We consider a system formed by two non-interaction parts, each part consists of a qubit A (or B) locally interacting respectively with a reservoir R_A (or R_B). The qubit A and B is initially entangled. The total Hamiltonian of the two qubits and the reservoirs can thus be written as follows [14,15]:

$$\hat{H} = \frac{1}{2}\omega_{0A}\sigma_A^z + \frac{1}{2}\omega_{0B}\sigma_B^z + \sum_k \omega_k^A a_k^+ a_k + \sum_j \omega_j^B b_k^+ b_k + \sum_{n=A,B}(\sigma_n^+ R_n + \sigma_n^- R_n^+), \tag{1}$$

where $a_k(a_k^+)$ is the annihilation (creation) operator of the kth mode (of frequency ω_k) of the reservoir interacting with the first subsystem A. Similarly $b_j(b_j^+)$ is the annihilation (creation) operator of the jth mode (of frequency ω_j) of the reservoir interacting with the second subsystem B. Dynamics of each part, consisting of a qubit with excited state $|1\rangle$ and ground state $|0\rangle$ which is coupled to a reservoir of field modes that initially in the vacuum state, can be represented by the reduced density matrix in the qubit basis $\{|0\rangle \ \ |1\rangle\}$

$$\rho^k(t) = \begin{pmatrix} \rho_{00}^k(0)\left|h_k(t)\right|^2 & \rho_{01}^k(0)h_k(t) \\ \rho_{10}^k(0)h_k(t) & 1-\rho_{11}^k(0)\left|h_k(t)\right|^2 \end{pmatrix}, \tag{2}$$

$k = A, B$. The function $h_k(t)$ is defined as the solution of the equation

$$\frac{d}{dt}h_k(t) = -\int_0^t d\tau f_k(t-\tau)h_k(\tau). \tag{3}$$

where $f_k(t-\tau)$ [$f(t-\tau)$ for short] denotes the two-point reservoir correlation function which can be written as the Fourier transform of the spectral density $J(\omega)$

$$f(t-\tau) = \int_0^\infty d\omega J(\omega)e^{-i(\omega-\omega_0)(t-\tau)}. \tag{4}$$

We use the density matrix elements given in Eq. (2) to construct the reduced density matrix for the two-qubits system seen in Appendix [16].

The entanglement of the bipartite system should be quantified by the concurrence [17]. In X-state, the concurrence can be easily calculated by using [18]

$$C(t) = 2\max\left\{0, \left|\rho_{14}(t)\right| - \sqrt{\rho_{22}(t)\rho_{33}(t)}, \left|\rho_{23}(t)\right| - \sqrt{\rho_{11}(t)\rho_{44}(t)}\right\}. \tag{5}$$

In the following section, we analyze the effect of independent amplitude damping channel on the dynamics of entangled quantum systems. We consider that the spectral density of each dissipative channel has the following Lorentzian distribution:

$$J_k(\omega) = \frac{1}{2\pi}\frac{\gamma_k\lambda_k^2}{(\omega_{0k}-\delta_k-\omega)^2+\lambda_k^2}, \tag{6}$$

$k = A, B$. γ_k is system-environment coupling strength; λ_k, defining the spectral width of the coupling, is connected to the reservoir correlation time τ_R by the relation $\tau_{Rk} = \lambda_k^{-1}$; a smaller λ_k indicates a longer correlation time and hence, more significant non-Markovianity. $\delta_k = \omega_{0k} - \omega_c$ is the detuning between ω_c and ω_{0k}, and ω_c is the center frequency of the cavity. It is worth noting that the effective coupling between the qubit and environment decreases when the detuning δ_k increases. According to the spectral density (6) in the non-Markovian process, the function $h_k(t)$ can be expressed as [19]

$$h_k^{(nM)}(t) = \exp\left[-\frac{(\lambda_k - i\delta_k)t}{2}\right] \times \left[\cos\ h\left(\frac{\Omega_k t}{2}\right) + \frac{\lambda_k - i\delta_k}{\Omega_k}\sin\ h\left(\frac{\Omega_k t}{2}\right)\right],$$

$$\Omega_k = \sqrt{(\lambda_k - i\delta_k)^2 - 2\gamma_k\lambda_k}. \tag{7}$$

It needs to illustrate that if $h_A = 1$ (or $h_B = 1$), information channel A (or B) will not have dissipation and we call it as single-sided damping channel; if $h_k \neq 1$, we call it as two-sided damping channel.

3. Entanglement dynamics of amplitude damping channel

3.1. Initial states of two qubit

Without losing the generality, in the following discussion, we consider two kinds of initial X-states. X-states contain the extended Werner-like states, defined as

$$\rho_\psi(r,\theta) = r\left|\psi(\varphi)\right\rangle\left\langle\psi(\varphi)\right| + \frac{1-r}{4}I_4,$$
$$\rho_\Phi(r,\theta) = r\left|\Phi(\theta)\right\rangle\left\langle\Phi(\theta)\right| + \frac{1-r}{4}I_4. \tag{8}$$

r is the purity of the initial states, which ranges from 0 for maximally mixed states to 1 for pure states, I_4 is the 4×4 identity matrix and

$$\left|\psi(\theta)\right\rangle = \cos\theta\left|00\right\rangle + \sin\theta\left|11\right\rangle,$$
$$\left|\Phi(\theta)\right\rangle = \cos\theta\left|01\right\rangle + \sin\theta\left|10\right\rangle,$$

is the Bell-like pure state, and the parameter θ is sometimes called as the degree of entanglement. From Eq. (5), we have

$$C_\psi(t) = \left|h_A(t)\right|\left|h_B(t)\right|\left\{r\left|\sin(2\theta)\right| - \frac{1}{2}\sqrt{m_A(t)m_B(t)}\right\}, \tag{9}$$

$$C_\Phi(t) = \left|h_A(t)\right|\left|h_B(t)\right|\left\{r\left|\sin(2\theta)\right| - \frac{1}{2}\sqrt{(1-r)n(t)}\right\}, \tag{10}$$

where

$$m_A(t) = 4r\sin^2\theta\left(1-\left|h_A(t)\right|^2\right) + (1-r)\left(2-\left|h_A(t)\right|^2\right),$$

$$m_B(t) = 4r\cos^2\theta\left(1-\left|h_B(t)\right|^2\right) + (1-r)\left(2-\left|h_B(t)\right|^2\right),$$

$$n(t) = 4 + (1-r)\left|h_A(t)\right|^2\left|h_B(t)\right|^2 - 2\left(\left|h_A(t)\right|^2 + \left|h_B(t)\right|^2\right) + 2r\cos\theta\left(\left|h_A(t)\right|^2 - \left|h_B(t)\right|^2\right)$$