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Correction of wave-front retrieval errors caused by the imperfect collimation of reference beam in phase-shifting interferometry

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ABSTRACT

In phase-shifting interferometry (PSI) the standard wave retrieval formulae are usually derived based on the condition of a plane reference beam normal to the recording plane. In practice, however, the reference wave may be a spherical wave of large radius or even have a slight inclination at the same time due to the imperfect collimation and coaxality of the optical system, and this fact will introduce phase distortion for the reconstructed object wave-front. A simple digital processing algorithm without any additional measurements is proposed to determine the unknown parameters of the spherical reference wave and then correct the object wave errors reconstructed with standard wave retrieval formulae. The effectiveness of this method is verified by a series of computer simulations, and its limitation is also discussed.

1. Introduction

On the contrary to traditional off-axis holography which requires special recording materials of very high resolution and wet processing, phase-shifting interferometry (PSI) can be realized with an on-axis configuration and a CCD camera as the recording device, and then has attracted much attention and been widely used in many fields with the rapid development of high-resolution CCD and digital image processing techniques recently [1,2]. A variety of algorithms of PSI with the use of three, four, five or more frames have been proposed [3–9]. Since in practice some measurement errors, for example, the phase step deviations, the camera non-linearity and the light source intensity fluctuation, are hard to be completely avoided, an extensive study on the effects of these error sources has been made and some error compensating algorithms have been suggested [6–11].

Usually, in deriving the wave retrieval formula for a certain algorithm in PSI, an assumption that the reference beam is a plane wave normal to the recording plane is used for the convenience both in theory and in practice. However, in real experiments, some derivation from this ideal assumption often occurs more or less due to the imperfect collimation of reference wave and the non-coaxality of the optical system. It means that the actual reference wave may be a slightly tilted plane wave, a spherical wave, or even a slightly tilted spherical wave. In these cases if we use the standard wave retrieval formulae designed for a normal plane reference wave, the calculated complex object wave in the recording plane P_H will naturally have a certain errors, and these errors will further cause the wave errors in original object plane P_O when the test object is placed in plane P_O, a distance away from P_H, and the wave field in P_O is digitally reconstructed from the wave field in P_H by an inverse Fresnel diffraction as happens in many applications [12–15]. These errors may yield totally mistaken phase maps in both planes P_O and P_H, and then lead to a significantly distorted topography of the test object surface.

In one of our previous works, we proposed a simple method without any additional measurements to correct the wave retrieval errors caused by the slight tilt of a plane reference beam [16]. In this paper, we will extend this method from a plane reference wave to a spherical reference wave. We will first explain its principles, then give a series of computer simulations for its verification, and finally have conclusions indicating its advantages and limitations.

2. Theoretical principles

A simplified scheme explaining the principle of PSI is given in Fig. 1, where P_O is the object plane and P_H the recording plane. The light field in plane P_O undergoes a Fresnel transform to reach plane P_H as object wave, denoted here as $u(x, y) = A_o(x, y)\exp[i\varphi_o(x, y)]$, and a reference wave with complex amplitude $u_r(x, y) = A_r(x, y)[i\varphi_r(x, y)]$ in plane P_H is introduced to interfere with the object

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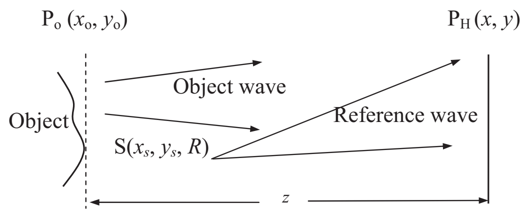

Fig. 1. A simplified scheme explaining the principle of PSI.

wave. Generally, the intensity distribution of the jth intereferogram can be expressed as

$$I_j = A_o^2 + A_r^2 + 2A_oA_r \cos\ (\varphi_o - \varphi_r - \delta_j), \tag{1}$$

where δ_j is the reference phase introduced by a phase shifter for jth frame. As an example, we consider the standard four-frame PSI with $\delta_1 = 0$, $\delta_2 = \pi/2$, $\delta_3 = \pi$ and $\delta_4 = 3\pi/2$. When the reference wave is exactly a plane beam normal to the plane P_H, we can obtain the standard formula for object wave retrieval in P_H [4],

$$u'(x, y) = \frac{1}{4A_r}[(I_1 - I_3) + i(I_2 - I_4)]. \tag{2}$$

However, if the phase distribution of reference wave in plane P_H, $\varphi_r(x, y)$, is not a constant (see Fig. 1), it is easy to show that the actual object wave in P_H should be [17,18]

$$u(x, y) = \frac{1}{4A_r} \exp\ [i\varphi_r(x, y)][(I_1 - I_3) + i(I_2 - I_4)]. \tag{3}$$

If we still use Eq. (2) to recover the object wave field in this case, the relation between the calculated object field u' and the actual field u is

$$u'(x, y) = \exp\left[-i\varphi_r(x, y)\right]u(x, y). \tag{4}$$

The most frequently happened collimation error of reference wave is that the reference beam is a spherical wave with a large radius R instead of an ideal plane wave; in this case Eq. (4) takes the form

$$u'(x, y) = \exp\left[-i\frac{\pi}{\lambda R}(x^2 + y^2)\right] u(x, y). \tag{5}$$

under paraxial approximation. Usually the parameter R is difficult to know in practice. In the following we will explain how to extract it directly from the intereferograms.

The discrete form of Eq. (5) is

$$u'(m, n) = \exp\left[-i\frac{\pi}{\lambda R}(m^2 + n^2)\right] u(m, n) \tag{6}$$

here m and n are the pixel indices standing for the discrete values of x and y, respectively. For convenience sake, here we suppose the ranges of m and n are from $-M$ to M and from $-N$ to N, respectively. In order to find the value of R, we introduce a new function

$$g(m, n) = \frac{u'(m + k, n + l)}{u'(m - k, n - l)} = c(m, n) \exp\left[-i\frac{4\pi}{\lambda R}(km + \mathrm{ln})\right] \tag{7}$$

where k and l are two preset small integers, and

$$c(m, n) = \frac{u(m + k, n + l)}{u(m - k, n - l)} \tag{8}$$

If the spatial change of $u(x, y)$ is not very fast, $c(m, n)$ may be considered approximately as a constant with absolute value of 1. Therefore, we may get an intensity distribution artificially

$$\begin{aligned} I(m, n) &= \left|\frac{1}{2}[g(m, n) + g(-m, -n)]\right|^2 \\ &= \frac{1}{2}\left\{1 + \cos\left[\frac{8\pi}{\lambda R}(km + \mathrm{ln})\right]\right\} \end{aligned} \tag{9}$$

Clearly there is a linear fringe pattern in $I(m, n)$. Noticing that in all the equations above the real coordinate of a pixel with a certain index such as m or n is actually the multiplication of the index and the pixel period in this direction, the spatial periods of this fringe pattern in x and y directions are

$$d_x = \frac{\lambda R}{4k\Delta x}, \quad d_y = \frac{\lambda R}{4l\Delta y}, \tag{10}$$

where k and l are index number, and Δx and Δy are the pixel periods in x and y directions, respectively. From these formulae and the measured periods we can decide R. Specifically, if we denote $d_x = p\Delta x$ and $d_y = q\Delta y$, where p is the pixel number in a fringe period in x direction and q is the one in y direction, we have

$$R = \frac{4kp(\Delta x)^2}{\lambda} = \frac{4lq(\Delta y)^2}{\lambda} \tag{11}$$

In practice, k and l can be chosen properly to change the measuring range of R and get the best fringe result for different object field. If one of k and l is zero, we get horizontal or vertical fringes, and one expression in Eq. (11) should be used. When both of k and l are not zero, we obtain oblique fringes, and the value R can be calculated as the average value of the two expressions.

An interesting fact about this algorithm is that it works well even for a slightly tilted spherical reference wave. If we suppose the reference wave is originated from the point (x_s, y_s, R) in this case, Eqs. (5) and (6) become

$$u'(x, y) = \exp\left[-i\frac{\pi}{\lambda R}(x^2 + y^2 - 2x_sx - 2y_sy)\right] u(x, y), \tag{12}$$

$$u'(m, n) = \exp\left[-i\frac{\pi}{\lambda R}(m^2 + n^2 - 2m_sm - 2n_sn)\right] u(m, n), \tag{13}$$

respectively. The similar calculations and assumption will lead to the same conclusions as expressed by Eqs. (9)–(11). In other words, the inclination of spherical reference wave has no effect on the extraction of its radius.

After the parameter R is found, we may further calculate the unknown x_s and y_s (m_s and n_s in discrete case) by constructing another intensity pattern

$$\begin{aligned} I'(m, n) &= \left|\frac{1}{2}[u'(m, n) + u'(-m, -n)]\right|^2 \\ &= \frac{1}{2}\left|u(m, n)\right|^2 \left\{1 + \cos\left[\frac{4\pi}{\lambda R}(m_sm + n_sn)\right]\right\}, \end{aligned} \tag{14}$$

in above derivation we have used an assumption $u(m, n) = u(-m, -n)$. Obviously, there is also a linear fringe pattern modulated by a field intensity. If the spatial periods of this fringe pattern in x and y directions are d_x' and d_y', respectively, we may obtain

$$x_s = \frac{\lambda R}{2d_x'}, \quad y_s = \frac{\lambda R}{2d_y'} \tag{15}$$

When all the parameters of reference wave, x_s, y_s and R, are determined, we can retrieve the correct object field in plane P_H from the result calculated by Eq. (2),

$$u(x, y) = \exp\left[i\frac{\pi}{\lambda R}(x^2 + y^2 - 2x_sx - 2y_sy)\right] u'(x, y) \tag{16}$$

And then the original object field $u(x_o, y_o)$ in plane P_O can be further computed by inverse Fresnel transform of $u(x, y)$.

3. Computer verification

Because the form of a real object surface is hard to know exactly, computer simulations have become a standard and most effective