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Effect of field of view on the accuracy of camera calibration

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ABSTRACT

A long focal length lens can improve the spatial resolution and capture more detailed information, so it has been considered for three-dimensional (3D) vision reconstruction. However, the field of view (FOV) will narrow with a long focal length lens. It is a general concept that it is extremely difficult to achieve high-accuracy calibration of a narrow FOV camera because of paraxial imaging. In this paper an in-depth study on this issue is conducted from the aspect of perspective deformation. First, the perspective deformation of a point is divided into three parts. Then, the noise immunity of each part under different FOVs is discussed to reveal the root cause of the difficulty in the calibration of narrow FOV cameras. It is found that the calibration accuracy could be generally maintained at the same level if the image noise is inversely proportional to the focal length. Simulations verify the correctness of the inferences of this study that are hoped to be helpful in overcoming this disadvantage of narrow FOV cameras.

1. Introduction

In the conventional field of stereo vision, a wide angle or normal lens is usually used due to a short working distance. However, images must be taken at a long distance as a result of the enlargement of the measured object and the complexity of the surroundings. In many applications, the spatial resolution will be too low to meet the accuracy requirement if a wide angle or normal lens is still used. Therefore, long focal length lenses are considered to be employed [1–3]. However, a long focal length lens will inevitably narrow the FOV, which means that the imaging beams will be closer to the optical axis. As a result, the error of camera calibration caused by image error (derived from corner detection error, imaging model error, control point error, etc [4]) will be significantly greater. Several measurement principles that are extensively adopted in photogrammetry, such as convergent imaging and orthogonal roll angles [3,5], are extremely important to the improvement of calibration accuracy. Nevertheless, they are insufficient to overcome the disadvantage of narrow FOV cameras. Additionally, the panoramic imaging technique has been proposed to try to achieve the equivalent expansion of the FOV [1,6]. However, there are usually still several unknown motion parameters that need to be determined and it is not easy to guarantee the motion constraint required by the panoramic imaging. The calibration of narrow FOV cameras is still an intractable problem. In many cases, camera parameters and coordinates of spatial points are determined simultaneously, and the accuracy of coordinate is the main focus of attention rather than the accuracy of calibration parameters. Even with large errors in calibration parameters, the coordinate accuracy may seem high as a consequence of interaction between camera parameters. The accuracy of calibration parameters is of our concern in this study.

Under the perspective imaging model, the perspective deformation is the basis for various calibration methods, such as calibration in computer vision where information of control points is usually precisely known [7,8], calibration in photogrammetry where information of control points is unnecessary [5,9], calibration based on scene features (e.g. parallelism and verticality) [10–12], etc. Therefore, in this study the perspective deformation is analyzed in detail and the reason why the calibration of a narrow FOV camera is so intractable is discussed taking two-dimensional (2D) template calibration as an example. Lens distortion is helpful in determining the principal point only when the center of lens distortion is considered identical to the principal point and the amount of lens distortion is much larger than the image noise [13]. Otherwise, lens distortion is useless for the determination of camera parameters except for lens distortion parameters. Consequently, the lens distortion is assumed to be zero in this study.

Section 2 briefly describes the perspective imaging model. Section 3 quantitatively analyzes the perspective deformation of a point by dividing it into three parts and the noise immunity of each part under different FOVs is discussed. Computer simulations provided in Section 4 verify the inferences of this study.

2. Perspective imaging model

The perspective imaging model can be expressed as $\boldsymbol{m} = \boldsymbol{A}(\boldsymbol{RX} + \boldsymbol{T})$, where $\boldsymbol{X}$ is the coordinate of an object point

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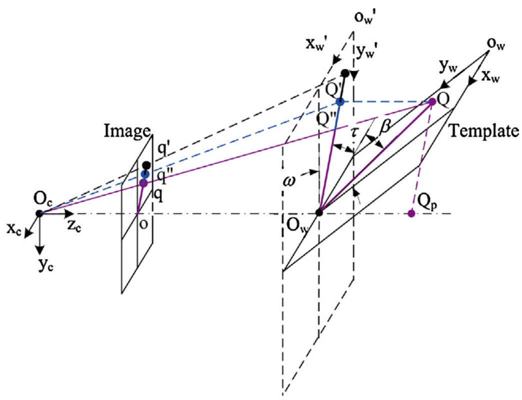

Fig. 1. Perspective imaging of a template point.

and $\boldsymbol{m}$ is the homogeneous coordinate of the image point. The intrinsic parameter matrix

$$\boldsymbol{A} = \begin{bmatrix} L_p/d_x & \gamma & u_0 \\ 0 & L_p/d_y & v_0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{1}$$

where L_p is the distance between the optical center and the image plane of the camera (i.e. the principal distance, which is always larger than the focal length unless the camera focuses at infinity), $d_x(d_y)$ is the distance on the image sensor between two horizontally (vertically) adjacent frame buffer picture elements [14], (u_0,v_0) is the coordinate of the intersection point of the optical axis and the image plane (i.e. the principal point), and γ describes the non-orthogonality of u and v axis.

$$\boldsymbol{R} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\varphi & 0 & \sin\varphi \\ 0 & 1 & 0 \\ -\sin\varphi & 0 & \cos\varphi \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\psi & -\sin\psi \\ 0 & \sin\psi & \cos\psi \end{bmatrix}, \tag{2}$$

and $\boldsymbol{T} = [t_x\ t_y\ t_z]^T$ are respectively the rotation and the translation matrix relating the object coordinate system to the camera coordinate system.

The intrinsic camera parameters to be calibrated include u_0, v_0, $a_u = L_p/d_x$, $a_v = L_p/d_y$ and γ. The extrinsic camera parameters include θ, φ, ψ, t_x, t_y and t_z.

3. Perspective deformation

3.1. Perspective deformation in a 2D template

As shown in Fig. 1, the intersection point O_w of the optical axis and the 2D calibration template is referred to the center point of the template, and its projection coincides with the principal point. A straight line in this template through O_w is referred to a center-line. The perspective deformation in a template is defined as the difference between the perspective image and the fronto-parallel image of the template (rotate the template around one center-line, the one parallel to the image plane, to make it parallel to the image plane. The perspective projection of the template in this position (see Fig. 1 $x'_w o'_w y'_w$) is referred to the fronto-parallel image).

To facilitate the analysis, the process of perspective projection of a template point is divided into two steps (see Fig. 1, taking point Q as an example). In the first step Q is orthogonally projected to the plane $x'_w o'_w y'_w$ getting the point Q'', and then Q'' is centrally projected to the image plane getting the image point q''. In the second step the line segment O_wQ'' is translated to the point Q getting the line segment Q_pQ, and then Q_pQ is centrally projected to the image plane getting the image point q. The image obtained in the first step is called the weak perspective image and the image obtained in the second step is the perspective image.

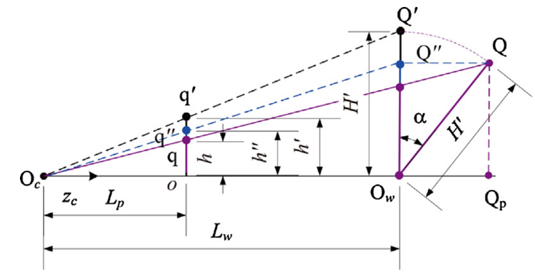

Fig. 2. Polar radius perspective deformation of a template point.

As shown in Fig. 1, the angle between the line segment O_wQ and the rotation axis (the center-line parallel to the image plane) is denoted by β, Thus, the angle between O_wQ'' and the rotation axis can be given by $\tau = \arctan(\tan\beta\cos\omega)$, where ω is the tilt angle of the template. β, ω, τ are all acute angles here. The difference in direction between oq and the projection of O_wQ in the fronto-parallel image satisfies

$$\Delta d = \tau - \beta \tag{3}$$

Δd is called the polar angle perspective deformation (PAPD) of point Q.

As shown in Fig. 2, in the fronto-parallel image the length of the projection of O_wQ can be expressed by $h' = L_pH'/L_w$, where L_w is the distance between the optical center O_c and the center point O_w, and H' is the length of O_wQ. If the angle $\angle QO_wQ''$ is denoted by α (α satisfies $\sin|\alpha| = \sin\omega\sin\beta$ with $-\pi/2 \le \alpha \le \pi/2$, and α is positive for the template points away from the camera and negative for the points close to the camera, separated by the rotation axis), then

$$h'' = \frac{L_pH'\cos\alpha}{L_w} \tag{4}$$

$$\Delta h_F = h'' - h' = \frac{L_pH'}{L_w}(\cos\alpha - 1) \tag{5}$$

h'' is the length of the projection of O_wQ''. Δh_F is called the first part of polar radius perspective deformation (FPRPD) of point Q. In the second step,

$$h = \frac{L_pH'\cos\alpha}{L_w + H'\sin\alpha} \tag{6}$$

$$\Delta h_S = h - h'' = \frac{-L_pH'^2\sin\alpha\cos\alpha}{L_w(L_w + H'\sin\alpha)} \tag{7}$$

h is the length of the projection of Q_pQ. Δh_S is called the second part of polar radius perspective deformation (SPRPD) of point Q.

According to the above definitions, we can see that the PAPD and the FPRPD are caused by template tilt while the SPRPD is caused by the difference in object distance. Actually, camera calibration is exactly the process of finding a set of camera parameters to make the perspective deformations presented in calibration images satisfy the perspective deformation formulas given above. It can be easily proved that the parameters of the perspective imaging model cannot be uniquely determined by the PAPD and the FPRPD, by which only some constraint equations on these parameters can be established. It is the existence of the SPRPD that makes the determination of all the model parameters possible. That is the reason