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Analytical description of the temperature field induced by laser heat source with any trajectory

Jerzy Winczek*, Agata Modrzycka, Elżbieta Gawrońska

Czestochowa University of Technology, Dabrowskiego 69, 42-200 Czestochowa, Poland

Abstract

The paper describes a three-dimensional temporary temperature field in the semi-infinite body caused by a moving laser heat source with any trajectory. In considerations the model of the surface heat source with the Gaussian distribution of power density was adopted. The trajectory of the source was approximated by straight sections. The calculation model of the temperature field takes into account changes in temperature caused by successive passes of the laser (temperature rise associated with the source action the source and cooling areas previously heated). Temperature field calculations in a rectangular steel member were carried out. The results of the calculations were illustrated by distributions of temporary and maximum temperature in a heated element.

Keywords: laser beam ; temperature field ; analytical solution

1. Introduction

Lasers as movable heat sources are commonly used in technology of metal and metal alloys processing, including cutting, heat treatment (quenching), welding, bending, and lately in additive manufacturing. Modelling of temperature field in these processes raises continued interest of many researchers, who created numerous works on this subject.

A review of mathematical models of laser cutting of steels in the early 90s of the last century has been presented in [1] by O'Neill and Steen. On the other hand Mackwood and Crafer in 2005 reviewed the literature [2] concerning laser welding and related processes, including more than 200 titles. Two approaches dominate in the modeling of the temperature field caused by a moving laser heat source: analytical (e.g. [3-7]) and numerical (e.g. [8-16]). Although in recent years FE method is most frequently used in the modeling of the temperature field, the analytical description is still very popular, because the analytical solution to heat conduction equation offers quicker assessment of temperature field and its dependence on parameters such as e.g. heat source velocity and power.

2. Analytical description of the temperature field

The starting point for the temperature field description in a homogeneous and isotropic body is the differential equation of heat conduction based on the energy conservation law [17]:

$$k\nabla^2 T(\mathbf{r},t) = C_p \rho \frac{\partial T(\mathbf{r},t)}{\partial t} - \frac{\partial Q_v}{\partial t} \tag{1}$$

* Corresponding author. Tel.:+48-34-3250-634; fax: +48-34-3250-509.
E-mail address: winczek@imipkm.pcz.czest.pl

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Nomenclature

a	thermal diffusivity (m^2/ s)
C_p	specific heat (J/kg K)
k	thermal conductivity (W/m K)
l_i	length of *i-th* segment (m)
q	heat source (J)
t	time (s)
t_j	time of heating of *j*-th segment
$T(x,y,z,t)$	temperature of body (K)
T_0	initial temperature of body (K)
t_0	quantity characterized surface heat source distribution (s)
t_{pi}	auxiliary time after *i*-th weld (s)
v	velocity of heat source (m/s)
x, y, z	global Cartesian coordinates

x', y', z' - coordinates of the heat source (m)
$x_{0i}, y_{0i}, x_{ki}, y_{ki}$, - coordinates of *i-th* segment beginning and ending (m)

Greek letters
β - angle between direction of heat source motion and axis x (rad)
ρ - density (kg/m^3)

which after the introduction of temperature compensation coefficient:

$$a = \frac{k}{C_p \rho} \tag{2}$$

assumes the form:

$$a\nabla^2 T(\mathbf{r},t) = \frac{\partial T(\mathbf{r},t)}{\partial t} - \frac{1}{C_p \rho}\frac{\partial Q_v}{\partial t} \tag{3}$$

where k denotes thermal conductivity (W/m K), a - thermal diffusivity (m^2/ s), C_p – specific heat (J/kg K), ρ - density (kg/m^3). When searching for the temperature field description, a solution of the basic equation is generally used (1) for the infinite body with a momentary/temporary heat source Q applied at any point on the body, with the coordinates (x, y, z) [18 - 23]:

$$T(x,y,z,t) = \frac{q}{C_p \rho (4\pi a t)^{3/2}} \exp\left(-\frac{(x-x')^2 + (y-y')^2 + (z-z')^2}{4at}\right) \tag{4}$$

Solution of an equation for the model of a massive body (half-infinite) (1) has the following form:

$$T(x,y,z,t) = \frac{2q}{C_p \rho (4\pi a t)^{3/2}} \exp\left(-\frac{(x-x')^2 + (y-y')^2 + (z-z')^2}{4at}\right) \tag{5}$$

with the following initial-boundary conditions:

$$\begin{cases} T(x,y,z,0) = T_0 \\ \dfrac{\partial T}{\partial x} = \dfrac{\partial T}{\partial y} = 0 \ \text{ for } \ \mathrm{x, y} \to \pm\infty \\ \dfrac{\partial T}{\partial z} = 0 \ \text{ for } \ \mathrm{z} \to \pm\infty \\ \left.\dfrac{\partial T}{\partial z}\right|_{z=0} = 0 \end{cases} \tag{6}$$