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Geometrical Method for Increasing Precision of Machine Building Parts

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Abstract

Calculation of complex curves with high accuracy is challenging. The curve is approximated by segments of straight lines and carried out smoothing. A new method of analytic geometry to calculate the trajectories of mechatronic systems and CAD/CAM is offered. The theoretical basis of the method is symmetries in the Euclidean plane. The method can accurately calculate the trajectory for the centrally symmetric conic sections and, in some cases, arbitrary differentiable planar curves. The method gives an accurate analytical solution without using radicals. This allows you to find the formulaic dependencies for further calculations. The existence of Lissajous figures allows assuming the presence of a universal method of calculation of complex trajectories based on planar differentiable curves. Some examples of the design of the kinematic mechanisms are presents additionally.

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Keywords: mechatronic systems; calculation of trajectory; symmetries; arbitrary curves.

1. Introduction

Precision manufacturing of engineering products is one of the important tasks. Calculation of complex curves with high accuracy is challenging. The curve is approximated by segments of straight lines and carried out smoothing. Curves and surfaces are sleek by splines methods adequately, but the process is not exact by definition. The solution is offered in the field of analytical geometry. Thus we solve the problem of the uniquely transfer of geometric models between CAD/CAM systems and enterprises additionally. Superposition of simple kinematic links gives rise to a complex mechatronic system. The problem of path accuracy is manifested in the design of metal-cutting equipment is particularly acute. Even flat trajectory of the mechanism in the Euclidean plane is a complex curve [1,2,3]. Jordan curve parameters in the analytic representation are difficult to find. To do this, you must solve the characteristic equation , where – transformation matrix, – scalars, – vector.

Equation can be solved by modern mathematical methods using projective transformations in spaces with lots of measurements [4]. Research is focus in the field of topology, so use them for engineering calculations is difficult. Convex geometry is close to analytic geometry greatest. Important practical results are obtained from its use. She is formulated by adding the ZF-axioms of set theory. The main objective of the reported studies getting sensible results in mechatronic systems and CAD/CAM. Codd's relational algebra as a practical part of ZF-theory has been applied additionally. Let's look at the theory of motion of a curves point in the Euclidean plane initially.

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2. Extended table of Diuedonne's symmetries

Hilbert formulating axioms for Euclidean plane suggested that must be considered construction of linguistic rules. We obtain the Euclidean plane as text by analogy with semiotic analyze of drawings according Leibniz's method of similarity. Levels of study of the text are internal relations in plane. As basic postulates were used: permutation, mirror and with unitary matrix symmetries by Dieudonne [5]; table automorphisms and transfer symmetry by H. Weyl; definition of symmetry by M. Born and binary automorphisms by F. Bachman.

Main attention of research was devoted on permutation symmetry [6, 7]. The Euclidean plane is relation table. The proof is easy, because relation algebra may be working with finite and indefinite table. The conduct of symmetry was considered by relation algebra and semiotic analyze. Application of the method to the Dieudonne automorphisms show that binary symmetry belongs to two mathematical disciplines: set theory and universal algebra.

Extended table of Dieudonne symmetries was built on the base of knowledge symmetry and relational algebra:

- Existing of set ($A \neq \varnothing$ Zermelo)
- Existing of relation ($a_1 R a_2$ Codd)
- Membership element of set ($a \in A$ Fraenkel)
- Universal relation ($f : \Omega \to \Omega'$ implication)
- Linguistic description of the set
- Linguistic presentation of the relation
- Saving cardinality ($\mathbf{m}(A) = const$ Lagrange)
- Saving power relations ($n = const$ in $C_1 x^n + C_2 y^n + C_3 x^{n-1} y^{n-1} + \ldots + C_{k-1} x + C_k y + C_0$ Klein)
- Linguistic order ($X \neq Y$ Descartes)
- Mathematical order ($a_i \prec a_{i+1}$, where $a_i, a_{i+1} \in \mathbf{R}$ Kantor).
- Permutation ($a_i \leftrightarrow a_j$)
- Mirror ($a_i \bullet -1 = -a_i$)

Since the connection between automorphisms 10 and 12 first opened Gilbert, the symmetry of knowledge may be named in his honor. Symmetry with numbers 1 and 2 is combined $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. Symmetries are grouped in three structured stages: symmetries existence; properties of automorphisms; numeric symmetry. Floor of existence consists the levels the antinomies and the main properties of the object. Stage of attributes include levels of representation in linguistic systems and cardinalities. Numeric floor consists of a order description and the classical symmetries. Gilbert opened the structural properties of symmetry, and Euler's formula $e^{i\pi} = -1$ to determine the relationship between them in universal algebra. Symmetries are joined of the two sciences, so there are exist two methods for solving the characteristic equation.

3. Direct method of linear transformation for central symmetric conic sections

Elliptic and hyperbolic trajectory can be calculated using not projective transformations by the classical method [8] or direct analytical method [6]. Not projective solutions for Jordan curves are absent.

Let there be an arbitrary figure Φ – Jordan curve in the Euclidean plane $\mathbf{R}^2$ in a Cartesian coordinate system defined by the parametric equation

$$\begin{cases} x = f_x(t) \\ y = f_y(t) \end{cases}, \tag{1}$$

where $x, y, t \in \mathbf{R}$. Functions $f_x(t)$ and $f_y(t)$ are piecewise continuous. If the equation's figure is defined $y = f(x)$, it is always possible to write $\begin{cases} x = t \\ y = f(t) \end{cases}$. Class of shapes defined only by the implicit function $F(x, y) = 0$ will not consider in this research. We carry out any transformation of the figure Φ defined by the matrix $\begin{pmatrix} a & h \\ g & b \end{pmatrix}$, where $a, b, h, g \in \mathbf{R}$. It is necessary to obtain the parameters of the transformed figure (to solve the characteristic equation).

Let consider the solution of the characteristic equation for the centrally symmetric conic sections. Only equation of own angle α take of the classic method [8]:

$$\tan 2\alpha = \frac{2(bh + ag)}{(a^2 + h^2) - (b^2 + g^2)}. \tag{2}$$

Parameters semiaxes considered difficult in the classical method, since they represent a radical dependence.

A new direct analytical method for the linear transformation was proposed earlier. He is free from radicals, so it is more simple and accessible for further mathematical derivations. The method is based on the permutation symmetry and other symmetries [7].

We calculate the angle β which is symmetrical own corner α for system (1)