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Response of single cell with acute angle exposed to an external electric field

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ABSTRACT

It is known that the electric field incurs effects on the living cells. Predicting the response of single cell or multilayer cells to induced alternative or static eclectic field has permanently been a challenge. In the present study a first order single cell with acute angle under the influence of external electric field is considered. The cell division stage or the special condition of reshaping is modelled with a cone being connected. In the case of cell divisions, anaphase, it can be considered with two cones that connected nose-to-nose. Each cone consists of two regions. The first is the membrane modelled with a superficial layer, and the second is cytoplasm at the core. A Laplace equation is written for this model and the distribution of its electric field is a sharp point in the single cell for which an acute angle model is calculated.

1. Introduction

Evaluation of responses of single cell organisms exposed to external electromagnetic fields in recently decade has increased significantly [4,5,10–13,18,23–25,29–31]. The information acquired from this response under electric and magnetic fields can play an important role in biology, biotechnology and medicine [3,6–8,14–17]. An external electric field can change the membrane voltage and impair endogenous membrane voltage [9,29,47]. In fact this sensitivity to applied external electric field leads to a significant increase of transmembrane voltage [9,18]. Electroporation phenomena that is very important in medicine and biology is one of the results of permeability changing as well as the voltage of a membrane which is induced by an applied external electric field [1,5,8,19–22].

In other hands the effects of electric fields on the cells and soft tissue is unclear yet [29–31]. Understanding the response of cells to DC and AC electric fields can help scientists and physicians to recognize normal cells and diseased [7,20]. Recently a lot of theoretical and experimental work has been done by scientist in this field. Zhou et al. have shown the RF-EMF with various specific absorption rate (SAR) at different time interval can affect to neurite growth of embryonic neural stem cells (eNSCs). They've shown the mRNA and protein expression of the proneural genes Ngn1 and NeuroD, which are crucial for neurite outgrowth, were decreased after RF-EMF exposure also. Their results show that the RF-EMF exposure impairs the potential adverse effects of neurons. They have shown the cultured vascular cells under DC electric field leads to migration, reorientation and elongation of them. EFs of 150–400 mV/mm play a role in the spatial organization of vascular structure [29]. They have concluded that endothelial cells derived from angiogenic micro-vascular as opposed to nonangiogenic macro-vascular tissues that are more responsive to electric stimulation [31].

Wound healing, treatment with low frequency of EMF is taken into consideration by. Many mechanisms have been proposed to describe how EF promotes wound closure also by other scientists [32–34,38,43]. Filipovic et al. have shown using EMF with specific frequency helps to control the cancer cells and colony in the breast cancer. Their computer modelling also predict such effects successfully however they suggest further studies before testing in humans [30]. The response of cells to electric field has been studied in many works separately. Some of these responses are known such as migration, transmission, reshape of cells, impair in endogenous electric field, polarization and electroporation [1,5,8,21,22,13,26–28,36,37,43,44]. Many kind of cells and tissues in an external EF is used to study. Osteoblasts and osteoclast [35], epidermal keratinocytes [28,36,37], corneal epithelial cells [38], astrocytes [39], endothelial cells [40], prostate cancer cells [41], chondrocytes [42], fibroblasts [43], dermal fibroblasts [44], Schwann cells [45] are exposed by AC and DC EFs to

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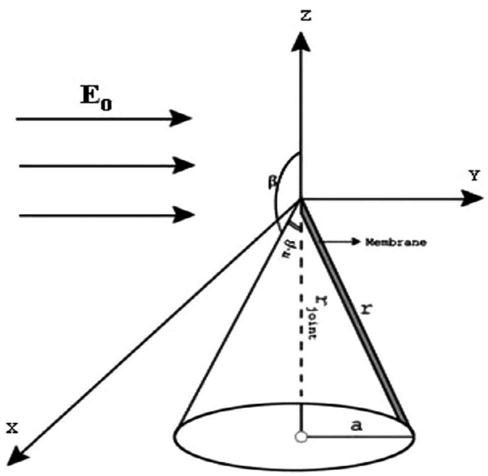

Fig. 1. Idealized representation of a single cell with acute angle in an external electric static field. *r*, is the radius of cell, r_{joint} is the effective radius between to conical cell in cone-to-cone mode.

study their effects. Differences between electromagnetics properties in normal and tumour cells can help us to control their behaviour under EMFs. Pulsed electromagnetic radiation, low frequency EMF alone or with combination of gamma or X-rays can improve the healing of cancer cells [6,17] and [46].

EMFs can damage the cells with changing the biological parameters like voltage, induced EMF strength, surface current, impairs in ionic gates, electric properties and other mechanism [47]. Physical cell modelling is a method to study the responds of cells to EMFs. Sardari et al. by using physical cell modelling were able to calculate the induced electric field. 103 V/cm as a threshold value for EF to damage the cells evaluated by them [46] that are comparable with experimental results that obtained by other scientists [6,48,49].

2. Materials and methods

In this novel approach we supposed that the cell membrane is homogenous. The charged membrane surface is neglected for this purpose. The numerical calculation is mandatory while in this case the analytical estimation is considered [9,13]. For an angled single cell without surface charge the voltage can be calculated by solving the Laplace equation. Consider an idealized model of a single cell with acute angle in an external electrostatic field which is shown in Fig. 1. The Laplace equation in general from is,

$$\nabla^2\varphi = 0 \tag{1}$$

where φ, is the induced EF potential. A standard solution of Laplace's equation in spherical coordinate system is,

$$\varphi(r,\theta) \cong Ar^{\upsilon}P_{\upsilon}(\cos\theta), \tag{2}$$

where, *A* is a coefficient which can be calculated from boundary conditions, υ is the order of Legendre equation and *P* is Legendre equation. To have a finite potential at origin, υ should be more than zero [2]. The electric fields for Eq. (2) are,

$$E_r = -\frac{\partial\varphi}{\partial r} \cong -\upsilon Ar^{\upsilon-1}P_{\upsilon}(\cos\theta) \tag{3}$$

$$E_{\theta} = -\frac{\partial\varphi}{\partial\theta} \cong Ar^{\upsilon-1}\sin\theta.P'_{\upsilon}(\cos\theta). \tag{4}$$

To find the answer of a single cell under the influence of an applied electric field with notice to Fig. 1 and with consideration of the geometry, the Bessel's function is good point to start,

$$J_{\upsilon}(x) = \left(\frac{x}{2}\right)^{\upsilon}\sum_{j=0}^{\infty}\frac{(-1)^j}{j!\Gamma(j+\upsilon+1)}\left(\frac{x}{2}\right)^{2j} \tag{5}$$

where, Γ is Gamma function, if supposed $\upsilon = 0$ then it can be written,

$$J_0(x) = \sum_{j=0}^{\infty}\frac{(-1)^j}{j!j!}\left(\frac{x}{2}\right)^{2j}. \tag{6}$$

By changing the variable *x* to the flowing parameter,

$$x = (2\upsilon+1)\sin\frac{\theta}{2}. \tag{7}$$

Eq. (6) will change to,

$$J_0\left[(2\upsilon+1)\sin\frac{\theta}{2}\right] = \sum_{j=0}^{\infty}\frac{(-1)^j}{j!j!}\frac{1}{2^{2j}}\left(\sin\frac{\theta}{2}\right)^{2j}. \tag{8}$$

By using to the trigonometry relation and binomial expansion also,

$$\sin^2\frac{\theta}{2} = \frac{1}{2}(1-\cos\theta) \tag{9}$$

$$(1-\cos\theta)^j = \sum_{k=0}^{n}\binom{n}{k}\cos^k\theta. \tag{10}$$

Eq. (8) becomes to,

$$J_0\left[(2\upsilon+1)\sin\frac{\theta}{2}\right] = \sum_{j=0}^{\infty}\sum_{k=0}^{2j}\sum_{s=0}^{j}\frac{(-1)^j}{j!j!}\upsilon^k(-\cos\theta)^s \tag{11}$$

With notice to Eq. (2), the definition of a new variable,

$$\mu = \frac{1}{2}(1-x) \tag{12}$$

where $x = \cos\theta$, the Legendre function can be expand as a polynomial series like,

$$P_{\upsilon}(\mu) = 1 + \frac{(-\upsilon)(\upsilon+1)}{1!1!}\mu + \frac{(-\upsilon)(-\upsilon+1)(\upsilon+1)(\upsilon+2)}{2!2!}\mu^2 + \cdots \tag{13}$$

if $\upsilon = 0, 1, 2, \ldots,$ the standard functions of Legendre will obtain. If we rewrite Eq. (13) compendiously,

$$P_{\upsilon}(\cos\theta) \cong \sum_{j=0}^{\infty}(-1)^j\frac{\upsilon^{2j}}{j!j!}(\cos\theta)^j. \tag{14}$$

It is easy to find from Eqs. (10) and (13) which are equivalent, as well as from Eq. (11),

$$P_{\upsilon}(\cos\theta) \cong J_0\left[(2\upsilon+1)\sin\frac{\theta}{2}\right]. \tag{15}$$

The first root of Bessel's function happen at $x = 2.405$, [2].

$$(2\upsilon+1)\sin\frac{\theta}{2} = 2.405. \tag{16}$$

This equation, is valid for small θ. To apply Eq. (14) for single cell which is shown in Fig. 1 at sharp point, the geometry of Fig. 1 in small angles with respect to Eq. (16) will become,

$$\theta = \pi - \beta \quad \upsilon = \frac{2.405}{\pi-\beta} - \frac{1}{2}. \tag{17}$$

Eq. (17) and the order of Legendre's equation υ, for first zero of $P_{\upsilon}(\cos\beta)$ as a function of β, are plotted in Fig. 2. The best fitted line as an arbitrary approximated function for a large β and small υ is cleared, as the third curve which is:

$$\upsilon \cong \left[2ln\frac{2}{\pi-\beta}\right]^{-1}. \tag{18}$$

With regards to the geometry of the cell which is shown in Fig. 1, $\beta \to 0$, $\pi/2$ describes a conical hole (i.e. a sphere with a small hole