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Nonlocal strain gradient beam model for postbuckling and associated vibrational response of lipid supramolecular protein micro/nano-tubules

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A B S T R A C T

As a supramolecular construction, lipid protein micro/nano-tubules can be utilized in a variety of sustained biological delivery system. The high slenderness ratio of lipid tubules makes their hierarchical assembly into a desired architecture difficult. Therefore, an accurate prediction of mechanical behavior of lipid tubular is essential. The objective of this study is to capture size dependency in the postbuckling and vibrational response of the postbuckled lipid micro/nano-tubules more comprehensively. To this purpose, the nonlocal strain gradient elasticity theory is incorporated to the third-order shear deformation beam theory to develop an unconventional beam model. Hamilton's principle is put to use to establish the size-dependent governing differential equations of motion. After that, an improved perturbation technique in conjunction with Galerkin method is employed to obtain the nonlocal strain gradient load-frequency response and postbuckling stability curves of lipid micro/nano-tubules. It is revealed that by taking the nonlocal size effect into consideration, the influence of the type (geometrical parameters) of an axially compressed lipid micro/nano-tubule on its natural frequency in order decreases and increases within the prebuckling and postbuckling regimes. While the strain gradient size dependency plays an opposite role which causes that the influence of the type of lipid micro/nano-tubule on its natural frequency corresponding to the prebuckling and postbuckling domains increases and decreases, respectively.

1. Introduction

Inspired by the nature, micro- and nano-structures on various size scales could be manufactured through biomolecular self-assembly. The molecular self-assembly of lipids is one of the most useful supramolecular structures having a wide range of applications in real world. Meilander et al. [1] designed two different delivery systems to release compacted DNA using lipid micro/nano-tubules and agarose hydrogel. Zhou and Shimizu [2] analyzed lipid micro/nano-tubules as a self-assemble in liquid media having controllable diameters and length. Kameta et al. [3] used lipid tubules to release of oligo DNA and spherical proteins from organic nanotubes including pH and temperature effects.

In lipid-based micro/nano-tubular drug delivery systems, the micro/nano-tubule is subjected to various loading conditions as well as mechanical vibrations. Consequently, in order to design and produce more efficient and accurate systems, it is necessary to analyze the mechanical characteristic of a lipid supramolecular micro/nano-tubule. Some investigations have been carried out to demonstrate the biological importance for mechanical characteristics of lipid protein micro/nano-tubules. For instance, Rosso and Virga [4] presented an exact solution for the statics and dynamics responses of lipid micro/nano-tubules adhering to a plane wall. Frusawa et al. [5] reported a novel and simple method to align lipid micro/nano-tubules having moderate stiffness using micro-injection the aqueous dispersion. Zhao et al. [6] analyzed the bending and radial deformation of lipid micro/nano-tubules using atomic force microscopes. Also, Zhao et al. [7] studied the buckling instability of lipid micro/nano-tubules subjected to local radial atomic force indentations. Shen [8] constructed a nonlocal beam model for size-dependent nonlinear mechanical behavior of lipid micro/nano-tubules. Daneshmand et al. [9] developed a nonlocal shell model for the size-dependent dynamic behavior of protein microtubules including shear deformation effect. Zhong et al. [10] investigated numerically the nonlinear dynamic stability of lipid micro/nano-tubules on the basis of nonlocal beam model. Tadi Beni et al. [11] employed the couple stress theory for size-dependent buckling of protein microtubules.

Due to the lack of an internal material length scale parameter, modeling of nanostructures via conventional elasticity theory is insufficient to describe small scale effects. To elucidate this deficiency, a variety of non-classical continuum theories have been proposed, e.g.

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nonlocal elasticity theory [12], surface elasticity theory [13], modified strain gradient elasticity theory [14], and modified couple stress elasticity theory [15]. In recent years, several investigations have been carried out based upon these unconventional theories of elasticity to demonstrate different size effects on mechanical response of micro/nano-structures [16–34]. For instance, Romano et al. [35] introduced nonlocal integral constitutive laws for analysis of mechanical behaviors of elastic nanobeams. They also indicated that existence of a solution of nonlocal beam elastostatic problems is an exception, the rule being non-existence for problems of applicative interest [36]. Romano and Barretta [37,38] predicted a stiffer elastic response via a stress-driven integral nonlocal elasticity. After that, based on the newly developed nonlocal model, they analyzed the size-dependent vibration behavior of Euler–Bernoulli nanobeams [39].

Recently, Lim et al. [40] combined the nonlocal and strain gradient elasticity theories in order to add both nonlocality of higher-order stresses and nonlocal higher-order strain gradients. As a result, more reasonable explanation of size dependency in mechanical characteristics of nanostructures has been introduced incorporating two entirely different features of hardening-stiffness and softening-stiffness size effects simultaneously. Lately, some investigations have been carried out based on the nonlocal strain gradient elasticity theory. For instance, Li and Hu [41] reported the size-dependent critical buckling loads of nonlinear Euler–Bernoulli nanobeams based upon nonlocal strain gradient theory of elasticity. They also presented the size-dependent frequency of wave motion on fluid-conveying carbon nanotubes via nonlocal strain gradient theory [42]. Yang et al. [43] established a nonlocal strain gradient beam model to evaluate the critical voltages corresponding to pull-in instability of functionally graded carbon nanotube reinforced actuators at nanoscale. Simsek [44] used nonlocal strain gradient theory to capture the size effects on the nonlinear natural frequencies of functionally graded Euler-Bernoulli nanobeams. Farajpour et al. [45] proposed a new size-dependent plate model for buckling of orthotropic nanoplates based on nonlocal strain gradient elasticity theory. Tang et al. [46] studied the wave propagation in a viscoelastic carbon nanotube via nonlocal strain gradient elasticity theory. Li et al. [47] utilized the nonlocal strain gradient elasticity theory within the framework of the Euler-Bernoulli beam theory to explore bending, buckling and free vibration of axially functionally graded nanobeams. Lu et al. [48] analyzed the size-dependent free vibration response of shear deformable nanobeams via nonlocal strain gradient elasticity theory. Sahmani and Aghdam [49] put nonlocal strain gradient to use to predict size effects on the nonlinear instability of axially loaded microtubules embedded in the cytoplasm biomedium. They also constructed a nonlocal strain gradient shell model for post-buckling analysis of magneto-electro-elastic nanoshells [50].

The prime aim of the present investigation is to study the small scale effects on the postbuckling and free vibrations before and after the buckling point of axially compressed lipid micro/nano-tubules. To this end, two entirely different size effects including hardening-stiffness and softening-stiffness are taken into consideration via implementation of the nonlocal strain gradient theory of elasticity in the third-order shear deformable beam model. The non-classical governing differential equations of motion are established by Hamilton's principle. Then Galerkin method together with an improved perturbation technique is utilized to propose explicit analytical expressions for the nonlocal strain gradient postbuckling stability paths and associated natural frequency-deflection curves.

2. Nonlocal strain gradient beam model for lipid micro/nano-tubules

A $DC_{8,9}$ PC lipid micro/nano-tubule produced via twisting of bilayer stripe molecules is shown in Fig. 1. The molecules do not pack parallel to each other because of chirality. Pinned boundary condition is supposed for the both ends of the lipid micro/nano-tubule having length of L, radius R and thickness h. The coordinate system is attached to the lipid tubule in such a way that x-axis is assumed along the tubule longitudinal axis, and z-axis is along tubule thickness.

Within the framework of the third-order shear deformation beam model, the components of displacement field along different coordinate directions can be expressed as

$$\begin{aligned} u_x(x,z,t) &= u(x,t) + z\psi(x,t) - \frac{4z^3}{3h^2}\left(\psi(x,t) + \frac{\partial w(x,t)}{\partial x}\right) \\ u_y(x,z,t) &= 0 \\ u_z(x,z,t) &= w(x,t) \end{aligned} \tag{1}$$

in which u, v and w denote the displacement components of the lipid micro/nano-tubule along x-, y- and z-axis, respectively. Also, ψ is the rotation relevant to the cross section of the lipid tubule at neutral plane normal about y-axis.

Consequently, the non-zero strain components are derived as

$$\varepsilon_{xx} = \varepsilon_{xx}^0 + z(\kappa_{xx}^{(0)} + z^2\kappa_{xx}^{(2)}) = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 + z\frac{\partial \psi}{\partial x} - \frac{4z^3}{3h^2}\left(\frac{\partial \psi}{\partial x} + \frac{\partial^2 w}{\partial x^2}\right) \tag{2a}$$

$$\gamma_{xz} = \gamma_{xz}^0 + z^2\kappa_{xz}^{(2)} = \psi + \frac{\partial w}{\partial x} - \frac{4z^2}{h^2}\left(\psi + \frac{\partial w}{\partial x}\right) \tag{2b}$$

where ε_{xx}^0, γ_{xz}^0 represent the mid-plane strain components, $\kappa_{xx}^{(0)}$ is the first-order curvature component, and $\kappa_{xx}^{(2)}$, $\kappa_{xz}^{(2)}$ are the higher-order curvature components.

It has been previously demonstrated that the size dependency in mechanical behavior of nanostructures has two entirely different effects which may lead to stiffen or soften the structure. Motivated by this fact, Lim et al. [40] proposed a more comprehensive unconventional continuum theory namely as nonlocal strain gradient elasticity theory which contains the both nonlocal and strain gradient size effects simultaneously. Thereby, the total nonlocal strain gradient stress tensor Λ for a beam-type structure can be read as below [40]

$$\Lambda_{xx} = \sigma_{xx} - \frac{\partial \sigma_{xx}^*}{\partial x} \tag{3a}$$

$$\Lambda_{xz} = \sigma_{xz} - \frac{\partial \sigma_{xz}^*}{\partial x} \tag{3b}$$

where σ and σ^* are, respectively, the stress and higher-order stress tensors which can be defined as

$$\sigma_{ij} = \int_\Omega \{\varrho_1(|\mathscr{X}' - \mathscr{X}|)C_{ijkl}\varepsilon_{kl}(\mathscr{X}')\}d\Omega \tag{4a}$$

$$\sigma_{ij}^* = l^2 \int_\Omega \left\{\varrho_2(|\mathscr{X}' - \mathscr{X}|)C_{ijkl}\frac{\partial \varepsilon_{kl}(\mathscr{X}')}{\partial x}\right\}d\Omega \tag{4b}$$

in which C stands for the stiffness matrix, $\varrho_1$and ϱ_2 in order are the principal attenuation kernel function including the nonlocality and the additional kernel function associated with the nonlocality effect of the first-order strain gradient field, $\mathscr{X}$ and $\mathscr{X}'$ denote, respectively, a point and any point else in the body, and l stands for the internal strain gradient length scale parameter. Following the method of Eringen, the constitutive relationship corresponding to the total nonlocal strain gradient stress tensor of a beam-type structure can be given as

$$\left(1 - \mu^2\frac{\partial^2}{\partial x^2}\right)\Lambda_{ij} = C_{ijkl}\varepsilon_{kl} - l^2C_{ijkl}\frac{\partial^2\varepsilon_{kl}}{\partial x^2} \tag{5}$$

where μ is the nonlocal parameter. As a result, the nonlocal strain gradient constitutive equation for a lipid micro/nano-tubule can be written as

$$\left(1 - \mu^2\frac{\partial^2}{\partial x^2}\right)\begin{Bmatrix}\sigma_{xx}\\ \sigma_{xz}\end{Bmatrix} = \left(1 - l^2\frac{\partial^2}{\partial x^2}\right)\begin{bmatrix}Q_{11} & 0\\ 0 & Q_{44}\end{bmatrix}\begin{Bmatrix}\varepsilon_{xx}\\ \gamma_{xz}\end{Bmatrix} \tag{6}$$