Journal of Hydrology 565 (2018) 224–236

Contents lists available at ScienceDirect

Journal of Hydrology

journal homepage: www.elsevier.com/locate/jhydrol

Research papers

Environmental transport in wetland channel with rectangular cross-section: Analytical solution by Chatwin's asymptotic expansion

J.L. Guo[a,1], W.Q. Jiang[a,1], L. Zeng[b,c], G.Q. Chen[a,*]

[a] College of Engineering, Peking University, Beijing 100871, China
[b] State Key Laboratory of Simulation and Regulation of Water Cycle in River Basin, China
[c] Institute of Water Resources and Hydropower Research, Beijing 100038, China

ARTICLE INFO

This manuscript was handled by G. Syme, Editor-in-Chief

Keywords:
Spatial concentration distribution
Environmental dispersivity
Environmental transport
Wetland flow
Taylor dispersion

ABSTRACT

Predicting the evolution of concentration distribution of environmental transport in wetland flows has a broad range of applications in ecological engineering practice. By extending a previous study, the spatial concentration distribution of solute transport due to an instantaneous environmental emission in a wetland channel with rectangular cross-section is presented in this work by means of Chatwin's long-time asymptotic technique. To account for the effect of high order moment, skewness of the transverse mean concentration distribution is analyzed. Two important parameters, i.e., aspect ratio (depth to width) and damping factor, on the dispersion process are discussed. Four studied cases illustrated that the transverse concentration distribution is nonuniform even at large time and the aspect ratio can affect the transverse concentration distribution greatly. The cross-sectional mean concentration distribution is asymmetrical in the initial stage and shows an asymptotic Gaussian distribution at large time. The dispersion process in a thick (width equals depth) wetland channel is quicker than that in a thin wetland channel. The spatial concentration distribution is dominated by the lateral-sidewall-effect on a large time scale for environmental transport in a thin wetland channel.

1. Introduction

Wetlands have received increasing attention, due to their essential ecosystem services, such as pollutants filtration, flood prevention, biodiversity maintenance and groundwater recharge, etc. (Costanza et al., 1989, 1997; Mitsch and Gosselink, 2015). With the acceleration of industrialization and urbanization, large amount of industrial and domestic sewage is discharged into the wetland (Ouyang and Guo, 2018; Zhang et al., 2009; Oberholster et al., 2008; Zann, 2000). Excess emission of pollutants pose a threat to the ecological carrying capacity of the wetland (Houser and Richardson, 2010). For ecological risk assessment and wastewater treatment engineering associated with the wetlands, concentration distribution of the contaminant transport process is essential for predicting the influenced region where pollutant concentration is beyond some given standard levels (Fischer et al., 1979; Zeng et al., 2011; Zeng and Chen, 2009).

To describe the solute transport process in wetland flows, Taylor dispersion (Taylor, 1953, 1954a) is a fundamental concept, which refers to a stage of transport process when the cross-sectional mean concentration forms a Gaussian distribution after the initial injection of the solute. Since Taylor's pioneering research, extended efforts have been made to study the solute dispersion in environmental flows (open channels, rivers and estuaries), i.e. environmental dispersion (Li et al., 2017, 2018; Elder, 1959; Ng, 2000, 2006; Holley et al., 1970). For environmental transport in wetland channel, due to the existence of vegetation, the hydraulic processes of momentum and mass transport are complex, relating to multi-scales (i.e., stem-scale, canopy-scale and cross-section scale) (Nepf, 1999; Ghisalberti and Nepf, 2002; Zeng et al., 2011). Focusing on the dispersion effect in one main dimension of the cross-section of the wetland, much endeavors have been made to investigate the longitudinal dispersion coefficients. For example, Lightbody and Nepf (2006a,b) studied the dispersion process arising from stem-scale and depth-scale velocity heterogeneity in an emergent canopy. Murphy et al. (2007) investigated the longitudinal dispersion coefficient in the depth dimension in a two-zoned model with submerged plants. Huai et al. (2017) estimated the longitudinal dispersion coefficient under the lateral distribution of depth-averaged longitudinal velocity in a symmetric compound channel with emergent vegetation. Moreover, numerical simulation and laboratory measurement have been conducted to estimate the dispersion process in a vegetated

---

[*] Corresponding author.
E-mail address: gqchen@pku.edu.cn (G.Q. Chen).
[1] These authors contributed equally to this work.

https://doi.org/10.1016/j.jhydrol.2018.08.026
Received 7 June 2018; Received in revised form 10 August 2018; Accepted 11 August 2018

channel or with porous media (Katzourakis and Chrysikopoulos, 2018; Yan et al., 2017; Sonnenwald et al., 2017; Ghisalberti and Nepf, 2005).

For mathematical modelling, Zeng and Chen (2009) followed a basic approach to smear out the discontinuity between the two phases of water and solid in the wetland by the phase-average method (Rajagopal and Tao, 1995; Liu and Masliyah, 2005). Based on the phase-average model, a great deal of analytical studies on solute transport in wetland flows have been carried out. Focusing on the solute transport in the free-water-surface-effect dominated wetland flows, the vertical concentration distribution and depth-averaged concentration have been investigated (Wu et al., 2015; Jiang et al., 2017; Wang and Chen, 2017b; Wang and Huai, 2018). Moreover, the environmental dispersion in multi-layer (i.e., two-layer or three-layer) wetlands with free-water-surface-effect were also analyzed (Chen et al., 2012; Luo et al., 2017; Wang et al., 2013, 2014). For environmental dispersion in the lateral-sidewall-effect dominated wetland flows, Zeng et al. (2011, 2014) studied the width-averaged mean concentration distribution. Additionally, the environmental dispersion in two-zone (Wu et al., 2011; Chen, 2013) and three-zone (Wang et al., 2015; Luo et al., 2016) wetlands were also investigated.

Above analytical studies concentrated on the solute dispersion in a free-water-surface-effect dominated or lateral-sidewall-effect dominated wetland flow. However, the impact of vertical and lateral variations of the superficial velocity distribution on the dispersion process should be both taken into account when the vertical and lateral scales of the cross-section are comparable. In fact, for solute transport in three-dimensional channel flows, Doshi et al. (1978) first found that the longitudinal dispersivity in a rectangular conduit with vanishing aspect ratio is about eight times larger than that in the corresponding two-dimensional case without side-wall effect. Chatwin and Sullivan (1982) and Dutta et al. (2006) studied the longitudinal diffusivity in rectangular channels with different values of the aspect ratio. In addition, Pagitsas et al. (1986) analyzed the Taylor dispersion of solute in Poiseuille-type solvent flow occuring within a rectangular duct of small aspect ratio. In terms of the environmental dispersion in wetland channel with rectangular cross-section, Chen et al. (2010) have established a basic model and studied the cross-sectional superficial velocity distribution and the environmental dispersivity. Afterwards, the velocity profile and environmental dispersivity in a tidal wetland channel of rectangular cross-section were also analyzed (Zeng et al., 2012).

However, previous researches associated with three-dimensional wetland flows mainly studied the longitudinal dispersion coefficient and the cross-sectional mean concentration distribution. As to the spatial concentration distribution, little endeavor has been made. Actually, the transverse concentration distribution is far from uniform in the initial stage and even for a long time, which has been verified in the two-dimensional wetland flows (Wang and Chen, 2017b; Jiang et al., 2017). For some environmental and industrial applications, such as ecological risk assessment (Wu et al., 2015), what of primary interest is the influenced region where the concentration is beyond some given standard levels. Apparently, the influenced region is sensitive to the detailed concentration distribution, and just giving the transverse mean concentration might not be sufficient. Moreover, previous studies gave the cross-sectional mean concentration distribution by neglecting the effect of high order moments. The asymmetry of the cross-sectional mean concentration distribution in the initial time has not been concerned. Although the skewness of the cross-sectional mean concentration distribution in rectangular microfluidic flows has been investigated (Aminian et al., 2016, 2015), there is still a need to study the skewness of the transverse mean concentration distribution in three-dimensional wetland flows due to the presence of vegetation.

Therefore, by extending the previous study (Chen et al., 2010), this paper analytically studies the complete concentration distribution for environmental transport in a steady wetland channel with rectangular cross-section in consideration of the effect of high order moment by means of Chatwin (1970)'s long-time asymptotic expansion. The specific objectives of the present work are: (I) to illustrate the environmental dispersivity and skewness of the cross-sectional mean concentration distribution; (II) to present the evolution of the transverse mean concentration distribution, and (III) to depict the spatial concentration distribution of the environmental transport process. For practical applications, the impact of the aspect ratio (depth to width of the cross-section) and effect of the vegetation in flow on the dispersion process will be characterized.

2. Formulation for solute transport in wetland flow

By the phase-average technique, which can smear out the discontinuity between the two phases of water and solid in the wetland (Zeng and Chen, 2009), mass transport equation can be adopted as (Liu and Masliyah, 2005; Zeng and Chen, 2011)

$$\phi\frac{\partial C^*}{\partial t^*} + \nabla^*\cdot(\mathbf{u}^*C^*) = \nabla^*\cdot(\kappa\lambda^*\phi\nabla^*C^*) + \kappa\nabla^*\cdot(\mathbf{K}^*\cdot\nabla^*C^*), \tag{1}$$

where ϕ is porosity (dimensionless), C^* is the water phase based superficial concentration (kg m^{-3}), t^* is time (s), $\mathbf{u}^*$ is the superficial velocity (m s^{-1}), κ is tortuosity accounting for the spatial structure of aquatic plants (dimensionless), λ^* is mass diffusivity (m^2 s^{-1}), and $\mathbf{K}^*$ is mass dispersivity tensor (m^2 s^{-1}).

Consider concentration transport in a typical flow with constant ϕ, κ and $\mathbf{K}^*$ through a three-dimensional wetland channel with whole width $2W^*$ and depth H^*, in a right-handed Cartesian coordinate system with longitudinal x^*-axis aligned with the flow direction, lateral y^*-axis, vertical z^*-axis, and origin set as the lower left corner of the cross-section, as presented in Fig. 1. In the Cartesian coordinate system, $\mathbf{K}^*$ can be expressed as $\mathbf{K}^* = K^*_{x^*x^*}\mathbf{e}_{x^*}\mathbf{e}_{x^*} + K^*_{y^*y^*}\mathbf{e}_{y^*}\mathbf{e}_{y^*} + K^*_{z^*z^*}\mathbf{e}_{z^*}\mathbf{e}_{z^*}$. Then

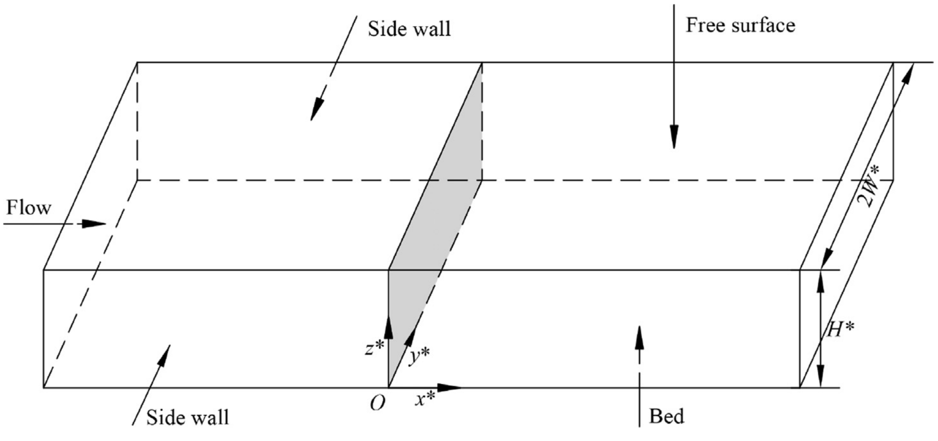

Fig. 1. Sketch of a wetland channel with rectangular cross-section.