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PRIMENESS PROPERTY FOR GRADED CENTRAL POLYNOMIALS OF VERBALLY PRIME ALGEBRAS

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ABSTRACT. Let F be an infinite field. The primeness property for central polynomials of $M_n(F)$ was established by A. Regev, i.e., if the product of two polynomials in distinct variables is central then each factor is also central. In this paper we consider the analogous property for $M_n(F)$ and determine, within the elementary gradings with commutative neutral component, the ones that satisfy this property, namely the crossed product gradings. Next we consider $M_n(R)$, where R admits a regular grading, with a grading such that $M_n(F)$ is a homogeneous subalgebra and provide sufficient conditions - satisfied by $M_n(E)$ with the trivial grading - to prove that $M_n(R)$ has the primeness property if $M_n(F)$ does. We also prove that the algebras $M_{a,b}(E)$ satisfy this property for ordinary central polynomials. Hence we conclude that, over a field of characteristic zero, every verbally prime algebra has the primeness property.

1. Introduction

The study of central polynomials is an important part of the theory of algebras with polynomial identites. Verbally prime algebras were introduced by A. Kemer [12] in his solution of the Specht problem and are of fundamental importance in the theory of p.i. algebras. The existence of central polynomials for verbally prime algebras was proved by Yu. P. Razmyslov [15] and earlier for matrix algebras, independently, by Formanek [11] and Razmyslov [14]. A p.i. algebra A is verbally prime if whenever $f(x_1, \ldots, x_r)$ and $g(x_{r+1}, \ldots, x_s)$ are two polynomials in distinct variables and $f \cdot g$ is an identity for A then either f or g is an identity for A. In his structure theory of T-ideals Kemer proves that over a field of characteristic zero every non-trivial verbally prime p.i. algebra is equivalent to one of the algebras $M_n(F)$, $M_n(E)$ (here E denotes the Grassmann algebra of a vector space of countable dimension) and certain subalgebras $M_{a,b}(E)$ of $M_{a+b}(E)$ (see Section 2).

Amitsur proved in [3] that if the field F is infinite the ideal of identities of $M_n(F)$ is a prime ideal. As an analog of Amitsur's result for central polynomials Regev proved in [16] the following primeness property on the central polynomials: if $f(x_1, \ldots, x_r)$ and $g(x_{r+1}, \ldots, x_s)$ are two polynomials in distinct variables and $f \cdot g$ is a central polynomial for $M_n(F)$ then both f and g are central.

Graded polynomial identities and graded central polynomials play an important role in the study of p.i. algebras, they were used in the theory developed by Kemer. Such identities are easier to describe in many important cases and they are related to the ordinary ones, for example, coincidence of graded identities implies the coincidence of the ordinary ones. In [9] regular gradings on the algebras $M_n(E)$

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