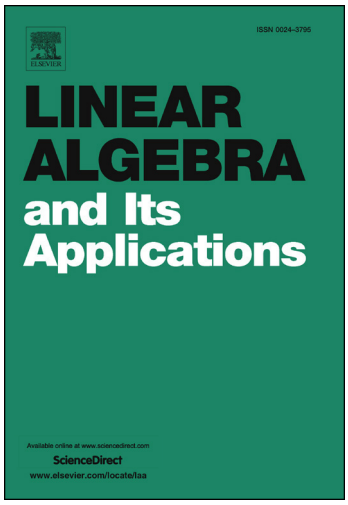

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Clustering behaviour in Markov chains with eigenvalues close to one

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Abstract

Finite, discrete, time-homogeneous Markov chains are frequently used as a simple mathematical model of real-world dynamical systems. In many such applications, an analysis of clustering behaviour in the states of the system is desirable, and it is well-known that the eigendecomposition of the transition matrix A of the Markov chain can provide such insight. Clustering methods based on the sign pattern in the second eigenvector of A are frequently used when A has dominant eigenvalues that are real. In this paper, we present a method to include an analysis for complex eigenvalues of A which are close to 1. Since a real spectrum is not guaranteed in most applications, this is a valuable result in the area of spectral clustering in Markov chains.

Keywords: Markov chain, clustering, complex eigenvalue
2010 MSC: 15B51, 60J10, 15A18

1. Introduction

A finite, discrete, time-homogeneous Markov chain refers to a mathematical model of a system which occupies, at any given time, one of a finite number of states $\{s_1, \ldots, s_n\}$ and transitions between states in discrete time-steps, according to prescribed transition probabilities. In particular, for any pair of states s_i and s_j, there is a given probability a_{ij} that the system moves to state s_j in one time-step, given that it is currently in state s_i. A Markov chain is memoryless, meaning that the movement of the system in the next time-step depends only on the current state the system occupies. A Markov chain can be

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