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Commuting functions of matrices over topological fields

Yaroslav Shitov

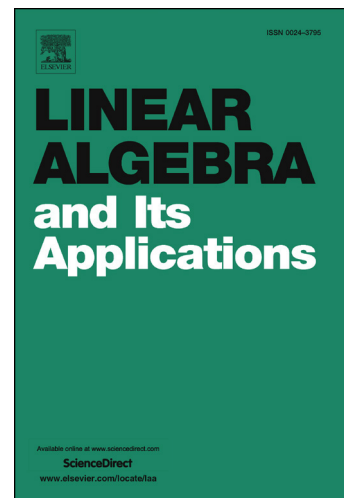
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Yaroslav Shitov

Russia, 129346 Moscow, Izumrudnaya ulitsa, dom 65, kvartira 4

Abstract

Any continuous function $f : \mathcal{F}^{n \times n} \rightarrow \mathcal{F}^{n \times n}$ satisfying $xf(x) \equiv f(x)x$ can be written as $f(x) \equiv \alpha_0(x)x^0 + \dots + \alpha_{n-1}(x)x^{n-1}$, where $\alpha : \mathcal{F}^{n \times n} \rightarrow \mathcal{F}^n$ is a mapping continuous on the set of non-derogatory matrices. Brešar and Šemrl proved this fact for $\mathcal{F} = \mathbb{C}$, and we give a short proof of their result which is moreover valid over any topological field. For $\mathcal{F} \in \{\mathbb{R}, \mathbb{C}\}$, we provide an example of a function f for which any appropriate α is discontinuous at every derogatory matrix.

Keywords: Commuting matrices, double centralizer theorem

2010 MSC: 16R60, 15A27

A field \mathcal{F} is called *topological* if it is endowed with a topology \mathcal{T} such that the addition, subtraction, multiplication, and division are continuous. (Here and in what follows, we assume that \mathcal{F}^n holds the natural *product topology*, and we call a mapping $f : X \rightarrow Y$ *continuous* if the preimage of any open subset of Y is open in X .) Also, we will assume that any singleton subset of \mathcal{F} is closed and not open; these assumptions are needed to get rid of the cases $\mathcal{T} = 2^{\mathcal{F}}$ and $\mathcal{T} = \{\emptyset, \mathcal{F}\}$, which are trivial to analyze. The obvious fact that every linear mapping on \mathcal{F}^n is continuous implies the following.

Observation 1. *An affine subspace $V \subsetneq \mathcal{F}^n$ is closed and not open.*

1. The result

We recall that a matrix $x \in \mathcal{F}^{n \times n}$ is called non-derogatory if the powers x^0, \dots, x^{n-1} are linearly independent over \mathcal{F} . In other words, there exists a tuple $\tau = (\tau_1, \dots, \tau_n)$ of indexes in $\{1, \dots, n\} \times \{1, \dots, n\}$ such that the

Email address: yaroslav-shitov@yandex.ru (Yaroslav Shitov)

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