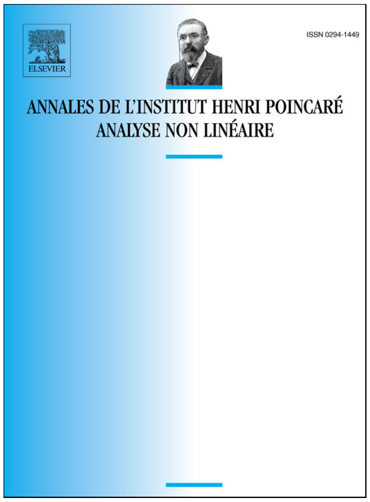

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PARTIAL DATA INVERSE PROBLEMS FOR MAXWELL EQUATIONS VIA CARLEMAN ESTIMATES

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Abstract. In this article we consider an inverse boundary value problem for the time-harmonic Maxwell equations. We show that the electromagnetic material parameters are determined by boundary measurements where part of the boundary data is measured on a possibly very small set. This is an extension of earlier scalar results of Bukhgeim-Uhlmann and Kenig-Sjöstrand-Uhlmann to the Maxwell system. The main contribution is to show that the Carleman estimate approach to scalar partial data inverse problems introduced in those works can be carried over to the Maxwell system.

Résumé. Dans cet article nous considérons un problème inverse aux limites pour les équations de Maxwell harmoniques en temps. Nous montrons que les paramètres électromagnètiques sont determinés par des mesures sur un très petit sous-ensemble du bord. Ces résultats pour le système de Maxwell sont une extension des résultats scalaires de Bukhgeim-Uhlmann et Kenig-Sjöstrand-Uhlmann. La contribution principale est de montrer que les methodes d'estimations de Carleman de ces articles peuvent être généralisées au système de Maxwell.

1. Introduction

In this paper we discuss an inverse problem for the time-harmonic Maxwell equations with partial data, and show that the electromagnetic material parameters are determined by measurements on certain parts of the boundary. The result is new even for the case of bounded domains in $\mathbb{R}^3$, but it will be convenient to formulate it more generally on compact manifolds with boundary.

Let (M, g) be a compact oriented Riemannian 3-manifold with C^∞ boundary. The electric and magnetic fields on M are described, respectively, by 1-forms E and H which satisfy the Maxwell equations in M:

$$\left\{ \begin{array}{l} *dE = i\omega\mu H, \\ *dH = -i\omega\varepsilon E. \end{array} \right. \tag{1.1}$$

Here $\omega > 0$ is a fixed frequency, d is the exterior derivative, and $*$ is the Hodge star operator for the metric g. The material parameters ε and μ are assumed to be complex valued functions in $C^3(M)$ and to satisfy

$$\mathrm{Re}(\varepsilon) > 0, \ \mathrm{Re}(\mu) > 0.$$

The inverse problem is formulated in terms of partial measurements of the boundary tangential traces tE and tH of E and H. Here, the tangential trace of a k-form η is

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