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Existence results for the radiation hydrodynamic equations with degenerate viscosity coefficients and vacuum

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Abstract

In this paper, we consider the compressible isentropic radiation hydrodynamic equations with density-dependent viscosity coefficients when the initial data are arbitrarily large and include vacuum at least appearing in the far field. Based on some reasonable assumptions for the radiation coefficients, we firstly establish the existence of a unique local regular solution, which implies the existence of the local strong solution. Moreover, we show a blow-up criterion for the regular solution that we obtained.

MSC: primary 35Q30, 35D35 ; secondary 35B44, 35K65

Keywords: Navier–Stokes–Boltzmann equations; Regular solutions; Strong solutions; Vacuum; Degenerate viscosity; Blow-up criterion

1. Introduction

The radiation hydrodynamic equations arise in high-temperature plasma [12] and in various astrophysical contexts [13]. The couplings between fluid field and radiation field involve momentum source and energy source depending on the specific radiation intensity driven by the

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so-called radiation transfer equation [21]. If the matter satisfies the local thermodynamical equilibrium (LTE), then the coupled system of Navier–Stokes–Boltzmann (RHD) equations has the following form:

$$\begin{cases} \frac{1}{c} I_t + \Omega \cdot \nabla I = A_r, \\ \rho_t + \operatorname{div}(\rho u) = 0, \\ \left(\rho u + \frac{1}{c^2} F_r\right)_t + \operatorname{div}(\rho u \otimes u + P_r) + \nabla P_m = \operatorname{div}\mathbb{T}, \end{cases} \tag{1.1}$$

where $t \geq 0$ is the time; $x \in \mathbb{R}^3$ is the spatial coordinate; $v \in \mathbb{R}^+$ is the frequency of photon; $\Omega \in S^2$ (S^2 is the unit sphere in $\mathbb{R}^3$) is the travel direction of photon; $I(v, \Omega, t, x)$ is the specific radiation intensity; $\rho(t, x)$ is the density; $u(t, x)$ is the velocity of the fluid; $P_m = A\rho^\gamma$ (A is a positive constant and γ is the adiabatic index.) is the material pressure; $\mathbb{T}$ is the stress tensor given by

$$\mathbb{T} = \mu(\rho)(\nabla u + (\nabla u)^\top) + \lambda(\rho)\operatorname{div}u\mathbb{I}_3, \tag{1.2}$$

where $\mathbb{I}_3$ is the 3×3 unit matrix, $\mu(\rho) = \alpha\rho$ is the shear viscosity, $\lambda(\rho) = \beta\rho$ is the second viscosity, where the constant α and β satisfy

$$\alpha > 0, \quad 2\alpha + 3\beta \geq 0. \tag{1.3}$$

The collision term A_r, the radiation flux F_r and the radiation pressure tensor P_r are given by

$$A_r = S - \sigma_a I + \int_0^\infty \int_{S^2} \left(\frac{v}{v'}\sigma_s I' - \sigma_s' I\right) \mathrm{d}\Omega' \mathrm{d}v',$$

$$F_r = \int_0^\infty \int_{S^2} I(v, \Omega, t, x)\Omega \mathrm{d}\Omega \mathrm{d}v, \;\; P_r = \frac{1}{c}\int_0^\infty \int_{S^2} I(v, \Omega, t, x)\Omega \otimes \Omega \mathrm{d}\Omega \mathrm{d}v,$$

where $I' = I(v', \Omega', t, x)$; $S = S(v, \Omega, t, x, \rho) \geq 0$ describes the rate of energy emission because of spontaneous process; $\sigma_a = \sigma_a(v, \Omega, t, x, \rho) \geq 0$ is the absorption coefficient; σ_s and σ_s' are the differential scattering coefficients with the form

$$\sigma_s = \sigma_s(v' \to v, \Omega' \cdot \Omega, \rho) = O(\rho), \quad \sigma_s' = \sigma_s(v \to v', \Omega \cdot \Omega', \rho) = O(\rho).$$

In this paper, we look for the local strong solution with the initial data

$$(I, \rho, u)|_{t=0} = (I_0(v, \Omega, x), \rho_0(x), u_0(x)), \quad (v, \Omega, x) \in \mathbb{R}^+ \times S^2 \times \mathbb{R}^3, \tag{1.4}$$

and the far field behavior

$$(I, \rho, u) \to (0, 0, 0) \quad \text{as} \quad |x| \to \infty. \tag{1.5}$$