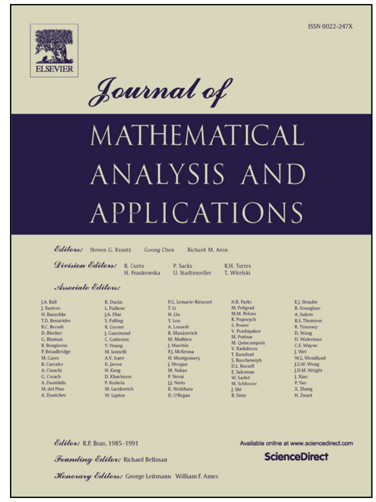

Inversion formulas and stability estimates of the wave operator on the hyperplane

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Inversion formulas and stability estimates of the wave operator on the hyperplane

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Abstract

One of the mathematical problems arising in Photoacoustic Tomography(PAT), a novel and promising technology in medical imaging, is to recover the initial function from the solution of the wave equation on a surface, where the detectors of PAT are located. We define the wave operator as the transform assigning to a given function f the solution of the wave equation on the detector surface (where the detectors are located) with the initial function f. Here we study many properties of this wave operator including the inversion formulas and stability estimates assuming that the detector surface is a hyperplane.

Keywords: Radon transform, photoacoustic, tomography, Spherical means
2010 MSC: 44A12, 92C55

1. Introduction

PhotoAcoustic Tomography (PAT) and Synthetic Aperture Radar (SAR) are based on the wave equation. One of the mathematical problems in PAT/TAT and SAR boils downs to recovering the initial function from the solution of the wave equation on a hyperplane. In other words, $p(\mathbf{x},t)$ satisfies the following initial value problem:

$$\begin{aligned} \partial_t^2 p(\mathbf{x},t) &= \triangle_{\mathbf{x}} p(\mathbf{x},t) \qquad (\mathbf{x},t)=(x_1,x_2,\cdots,x_n,t)\in\mathbb{R}^n\times(0,\infty),\\ p(\mathbf{x},0) &= f(\mathbf{x}) \\ \partial_t p(\mathbf{x},0) &= 0. \end{aligned} \tag{1}$$

Our data, the solution p is measured on the hyperplane $\{\mathbf{x}=(\boldsymbol{x},x_n)\in\mathbb{R}^n : x_n=0, \boldsymbol{x}\in\mathbb{R}^{n-1}\}$. In this case, one of our goals is to reconstruct f from $p((\boldsymbol{u},0),t)$ for $(\boldsymbol{u},t)\in\mathbb{R}^{n-1}\times[0,\infty)$. (An $n-1$-dimensional vector is denoted by a italic bold letter while an n-dimensional vector is denoted by a normal bold letter.)

For $f\in\mathcal{S}(\mathbb{R}^n)$, the Schwartz space, the solution of (1) is

$$p(\mathbf{x},t)=(2\pi)^{-n}\int\limits_{\mathbb{R}^{n-1}}\int\limits_{\mathbb{R}}\cos(t|(\boldsymbol{\xi},\zeta)|)e^{\mathrm{i}\mathbf{x}\cdot(\boldsymbol{\xi},\zeta)}\mathcal{F}f(\boldsymbol{\xi},\zeta)\mathrm{d}\zeta\mathrm{d}\boldsymbol{\xi},\quad \text{for}\quad (\mathbf{x},t)\in\mathbb{R}^n\times[0,\infty),$$

where $\mathcal{F}f$ is the Fourier transform of f and $(\boldsymbol{\xi},\zeta)$ is the dual variables to $(\boldsymbol{x},x_n)$ in the Fourier transform. Also, because of the initial conditions $p(\mathbf{x},0)=f(\mathbf{x})$ and $\partial_t p(\mathbf{x},0)=0$, we can allow the solution p to