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Sunghwan Moon

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Inversion formulas and stability estimates of the wave operator on the hyperplane

Sunghwan Moon

*Department of Mathematics, College of Natural Sciences
Kyungpook National University
Daegu 41566, Republic of Korea
sunghwan.moon@knu.ac.kr*

Abstract

One of the mathematical problems arising in Photoacoustic Tomography (PAT), a novel and promising technology in medical imaging, is to recover the initial function from the solution of the wave equation on a surface, where the detectors of PAT are located. We define the wave operator as the transform assigning to a given function f the solution of the wave equation on the detector surface (where the detectors are located) with the initial function f . Here we study many properties of this wave operator including the inversion formulas and stability estimates assuming that the detector surface is a hyperplane.

Keywords: Radon transform, photoacoustic, tomography, Spherical means
2010 MSC: 44A12, 92C55

1. Introduction

PhotoAcoustic Tomography (PAT) and Synthetic Aperture Radar (SAR) are based on the wave equation. One of the mathematical problems in PAT/TAT and SAR boils down to recovering the initial function from the solution of the wave equation on a hyperplane. In other words, $p(\mathbf{x}, t)$ satisfies the following initial value problem:

$$\begin{aligned} \partial_t^2 p(\mathbf{x}, t) &= \Delta_{\mathbf{x}} p(\mathbf{x}, t) & (\mathbf{x}, t) &= (x_1, x_2, \dots, x_n, t) \in \mathbb{R}^n \times (0, \infty), \\ p(\mathbf{x}, 0) &= f(\mathbf{x}) \\ \partial_t p(\mathbf{x}, 0) &= 0. \end{aligned} \tag{1}$$

Our data, the solution p is measured on the hyperplane $\{\mathbf{x} = (\mathbf{x}, x_n) \in \mathbb{R}^n : x_n = 0, \mathbf{x} \in \mathbb{R}^{n-1}\}$. In this case, one of our goals is to reconstruct f from $p((\mathbf{u}, 0), t)$ for $(\mathbf{u}, t) \in \mathbb{R}^{n-1} \times [0, \infty)$. (An $n-1$ -dimensional vector is denoted by a italic bold letter while an n -dimensional vector is denoted by a normal bold letter.)

For $f \in \mathcal{S}(\mathbb{R}^n)$, the Schwartz space, the solution of (1) is

$$p(\mathbf{x}, t) = (2\pi)^{-n} \int_{\mathbb{R}^{n-1}} \int_{\mathbb{R}} \cos(t|(\boldsymbol{\xi}, \zeta)|) e^{i\mathbf{x} \cdot (\boldsymbol{\xi}, \zeta)} \mathcal{F}f(\boldsymbol{\xi}, \zeta) d\zeta d\boldsymbol{\xi}, \quad \text{for } (\mathbf{x}, t) \in \mathbb{R}^n \times [0, \infty),$$

where $\mathcal{F}f$ is the Fourier transform of f and $(\boldsymbol{\xi}, \zeta)$ is the dual variables to (\mathbf{x}, x_n) in the Fourier transform. Also, because of the initial conditions $p(\mathbf{x}, 0) = f(\mathbf{x})$ and $\partial_t p(\mathbf{x}, 0) = 0$, we can allow the solution p to

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