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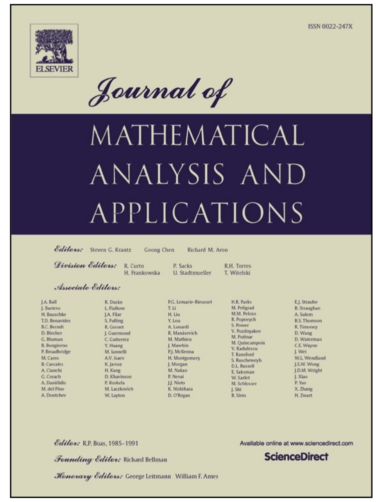

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Exponential Decay of A Thermoelastic System for A Thin Plate under Periodic Sunlight *

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Abstract

In this paper we investigate a thermoelastic system with Dirichlet or Neumann or natural boundary conditions. For completeness we derive the thermoelastic system model and show the stability results for different boundary conditions which are based on the analysis of the corresponding eigenvalue problems. Further we derive the equilibrium ambient temperature equations that admits a periodic solution modeling the daily heat source from the sun.

Keywords: Thermoelastic plate; Model; Stability

1 Introduction

We consider long time behavior of the dimensionless thermoelastic model

$$\begin{cases} w_{tt} + \Delta^2 w + \alpha\Delta\theta = 0 & \text{in } \Omega\times[0,\infty), \\ \theta_t - k\Delta\theta - \alpha\Delta w_t = -\gamma(t)\theta & \text{in } \Omega\times[0,\infty), \\ w(\cdot,0) = w_0(\cdot),\ w_t(\cdot,0) = w_1(\cdot), \theta(\cdot,0) = \theta_0(\cdot) & \text{in } \Omega, \end{cases} \tag{1.1}$$

subject to different sets of boundary conditions; here w is vertical deflection from the "middle surface" of a thin plate, θ is vertical temperature gradient of thin plate and $\gamma(t) = 4\sigma T^3(t)$, where $T(t)$ is the ambient temperature of the plate. Our model is slightly different from that in [1, 2] so we provide our detailed derivation in the next section. We would like to point out that θ here is not temperance but the temperature gradient in vertical direction, so the coefficient $\gamma(t) = 4\sigma T(t)^3$ arises from the Stefan-Boltzmann radiation Law [3, 4]

$$T_t = S(t) - \sigma T^4,$$

where $S(t)$ is the radiation received from the Sun. We assume that S is periodic and T reaches its dynamical equilibrium (see derivation in Appendix A). Hence, we assume that $\gamma(t)$ is a fixed function satisfying

$$\gamma(t) = \gamma(t+1) > 0 \quad \text{for every } t \geq 0. \tag{1.2}$$

We shall consider the following boundary conditions:

(1) Dirichlet Boundary Condition

$$\theta = w = \Delta w = 0 \quad \text{on } \partial\Omega\times[0,\infty). \tag{1.3}$$

Physically, this means that the boundary of the place is kept to the temperature of the environment and its edge is fixed.

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