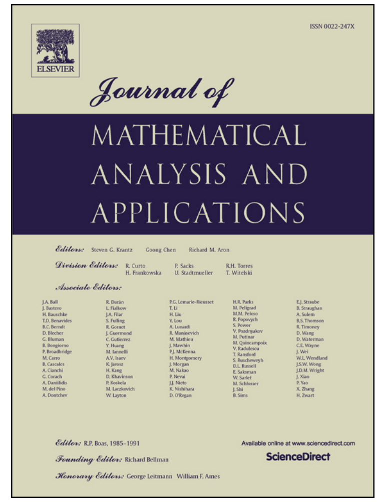

Spectrality of Moran measures with four-element digit sets

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SPECTRALITY OF MORAN MEASURES WITH FOUR-ELEMENT DIGIT SETS

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ABSTRACT. Let $\delta_E = \frac{1}{\#E}\sum_{a\in E}\delta_a$ denote the uniformly discrete probability measure on a finite set E. We prove that the infinite convolution (Moran measure)

$$\mu_{b,\{\mathcal{D}_k\}} = \delta_{b^{-1}\mathcal{D}_1} * \delta_{b^{-2}\mathcal{D}_2} * \cdots$$

admits an orthonormal basis of exponential provided that $\{\mathcal{D}_k\}_{k=1}^{\infty}$ is a uniformly bounded sequence of 4-digit spectral sets, $b = 2^{l+1}q$ with $q > 1$ an odd integer, and l sufficiently large (depends on $\mathcal{D}_k$). We also give some examples to illustrate the result.

1. Introduction

Let μ be a compactly supported Borel probability measure on $\mathbb{R}^d$. μ is called a *spectral measure* if there exists a countable set $\Lambda \subset \mathbb{R}^d$ such that $E(\Lambda) := \{e^{2\pi i<\lambda,x>} : \lambda \in \Lambda\}$ forms an orthonormal basis for $L^2(\mu)$. In this case, Λ is called a *spectrum* of μ and (μ, Λ) is called a *spectral pair*. If the normalized Lebesgue measure restricting on a Borel set Ω is a spectral measure, then Ω is called a *spectral set*. The study of spectral measures was first initiated by B.Fuglede in 1974 [8], who conjectured that $\Omega \subset \mathbb{R}^d$ *is a spectral set if and only if* Ω *is a translational tile*. The conjecture has been studied by many authors, e.g., Iosevich, Jorgensen, Kolountzakis, Laba, Lagarias, Matolcsi, Pedersen, Tao, Wang and many others ([17–23, 27, 28, 30, 34]), and it had baffled experts for 30 years until Tao [34] constructed the first counterexample, a spectral set which is not a tile on $\mathbb{R}^d$, $d \geq 5$. The example and technique were refined later to disprove the conjecture in both directions on $\mathbb{R}^d$ for $d \geq 3$. It is still open in dimensions $d = 1$ and $d = 2$. Despite the counterexamples, the exact relationship between spectral measures and tiling is still mysterious.

For non-atomic singular measures, a class of spectral measures was first found by Jorgensen and Pedersen (i.e., the $1/q$-Cantor measure where q is even) [19], and Strichartz supplemented their result with a simplified proof [31]. The result was

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