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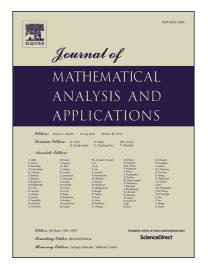
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SPECTRALITY OF MORAN MEASURES WITH FOUR-ELEMENT DIGIT SETS

MIN-WEI TANG AND FENG-LI YIN *

ABSTRACT. Let $\delta_E = \frac{1}{\#E} \sum_{a \in E} \delta_a$ denote the uniformly discrete probability measure on a finite set *E*. We prove that the infinite convolution (Moran measure)

$$\iota_{b,\{\mathcal{D}_k\}} = \delta_{b^{-1}\mathcal{D}_1} * \delta_{b^{-2}\mathcal{D}_2} * \cdots$$

admits an orthonormal basis of exponential provided that $\{\mathcal{D}_k\}_{k=1}^{\infty}$ is a uniformly bounded sequence of 4-digit spectral sets, $b = 2^{l+1}q$ with q > 1 an odd integer, and l sufficiently large (depends on \mathcal{D}_k). We also give some examples to illustrate the result.

1. INTRODUCTION

Let μ be a compactly supported Borel probability measure on \mathbb{R}^d . μ is called a *spectral measure* if there exists a countable set $\Lambda \subset \mathbb{R}^d$ such that $E(\Lambda) := \{e^{2\pi i \langle \lambda, x \rangle} : \lambda \in \Lambda\}$ forms an orthonormal basis for $L^2(\mu)$. In this case, Λ is called a *spectrum* of μ and (μ, Λ) is called a *spectral pair*. If the normalized Lebesgue measure restricting on a Borel set Ω is a spectral measure, then Ω is called a *spectral set*. The study of spectral measures was first initiated by B.Fuglede in 1974 [8], who conjectured that $\Omega \subset \mathbb{R}^d$ is a spectral set if and only if Ω is a translational tile. The conjecture has been studied by many authors, e.g., Iosevich, Jorgensen, Kolountzakis, Laba, Lagarias, Matolcsi, Pedersen, Tao, Wang and many others ([17–23,27,28,30,34]), and it had baffled experts for 30 years until Tao [34] constructed the first counterexample, a spectral set which is not a tile on \mathbb{R}^d , $d \geq 5$. The example and technique were refined later to disprove the conjecture in both directions on \mathbb{R}^d for $d \geq 3$. It is still open in dimensions d = 1 and d = 2. Despite the counterexamples, the exact relationship between spectral measures and tiling is still mysterious.

For non-atomic singular measures, a class of spectral measures was first found by Jorgensen and Pedersen (i.e., the 1/q-Cantor measure where q is even) [19], and Strichartz supplemented their result with a simplified proof [31]. The result was

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