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On (shape-)Wilf-equivalence for words

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Stankova and West showed that for any non-negative integer s and any permutation γ of $\{4, 5, \dots, s+3\}$ there are as many permutations that avoid 231γ as there are that avoid 312γ. We extend this result to the setting of words.
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1. Introduction

Let $\pi = \pi_1\pi_2\cdots\pi_n$ be a permutation of $\{1, 2, \ldots, n\}$ and $\sigma = \sigma_1\sigma_2\cdots\sigma_r$ be a permutation of $\{1, 2, \ldots, r\}$, $r \le n$. We say that the permutation π *contains the pattern* σ, if there are indices $1 \le i_1 < i_2 < \cdots < i_r \le n$ such that $\pi_{i_1}\pi_{i_2}\cdots\pi_{i_r}$ is in the same relative order as $\sigma_1\sigma_2\cdots\sigma_r$. Otherwise, π is said to *avoid the pattern* σ, or, alternatively, we say that π is σ*-avoiding*. As usual, we write S_n for the set of all permutations of $\{1, 2, \ldots, n\}$, and $S_n(\sigma)$ for the set of permutations in S_n that avoid σ.

The enumeration of permutations which avoid certain patterns has been a flourishing research subject since the seminal article [10] of Simion and Schmidt, where this research subject was "defined." The reader is referred to [8] and [4, Chapters 4 and 5] for in-depth accounts of the enumeration of pattern avoiding permutations.

We define the *direct sum* $\sigma \oplus \tau$ of two permutations $\sigma = \sigma_1\sigma_2\cdots\sigma_r \in S_r$ and $\tau = \tau_1\tau_2\cdots\tau_s \in S_s$ as

$$\sigma \oplus \tau = \sigma_1\sigma_2\cdots\sigma_r(\tau_1 + r)(\tau_2 + r)\cdots(\tau_s + r).$$

The starting point for the work on this paper was an email of Doron Zeilberger [12] to the second author saying[4]:

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According to http://en.wikipedia.org/wiki/Permutation_pattern,
Backelin, West & Xin (2007) proved that for any permutation
beta and any positive integer k, the permutations
12..k "+" beta and k....21 "+" beta are Wilf equivalent,
and in 2002 Stankova and West proved that 231 "+" beta and
312 "+" beta are Wilf equivalent.
According to Vince Vatter and Jonathan Bloom you have nice
proofs at least of the first result, in "Growth Diagrams, ...
Ferrers Shapes".
My main question is whether it is true, and whether there
exists a proof, of the above two results generalized to words
with a_1 1's, a_2 2's, ..., a_m m's, where the original case is
a_1 = ... = a_m = 1.
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Here, two patterns σ and τ are said to be *Wilf-equivalent*, denoted by $\sigma \sim \tau$, if $|S_n(\sigma)| = |S_n(\tau)|$ for all positive integers n. What Zeilberger refers to are the following two results.

Theorem 1 *([2, Theorem 2.1]). For all positive integers k, all non-negative integers s, and all patterns $\beta \in S_s$, the patterns $12\cdots k \oplus \beta$ and $k\cdots 21 \oplus \beta$ are Wilf-equivalent.*

[4] Notation is slightly adapted in order to conform with the notation of our paper.

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