Applied Mathematics and Computation 339 (2018) 556–567

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

A novel two-dimensional coupled lattice Boltzmann model for thermal incompressible flows

Yikun Wei [a,*], Hui Yang [a], Hua-Shu Dou [a], Zhe Lin [a,*], Zhengdao Wang [b], Yuehong Qian [c]

[a] The Province Key Laboratory of Fluid Transmission Technology, Faculty of Mechanical Engineering and Automation, Zhejiang Sci-Tech University, Zhejiang Province 310018 Hangzhou, China
[b] Shanghai Key Laboratory of Mechanics in Energy Engineering, Shanghai University, 200072 Shanghai, China
[c] School of mathematical science, Soochow University, 215006 Suzhou, China

ARTICLE INFO

Keywords:
Lattice Boltzmann method
Convection
Incompressible flows
Numerical simulation

ABSTRACT

A novel two-dimensional coupled lattice Boltzmann model is developed for thermal incompressible fluid flows. A modified equilibrium distribution function is proposed in the present model. A mesoscopic discrete force is coupled into the modified equilibrium distribution function based on the Boussinesq approximation. The outstanding advantages of the standard lattice Boltzmann method are retained in present model besides better numerical stability. The present model is validated by the numerical simulation of the natural and Rayleigh–Benard convection at a wide range of Rayleigh numbers. Excellent agreement between the present results and previous lattice Boltzmann method or theoretical prediction demonstrates that present model is an efficient numerical method for natural and Rayleigh–Bénard convection. Further, present model is also successfully assessed considering Rayleigh–Taylor instability. It is also easier and convenient to be implemented as compared with the previous thermal models.

1. Introduction

Lattice Boltzmann method (LBM) is a new numerical scheme for simulating viscous incompressible flows in the subsonic regime [1–8]. Instead of solving the usual continuum hydrodynamic equations, the LBM tries to model the fluid flow by tracking the evolution of the distribution functions of the microscopic fluid particles. This kinetic nature of the LBM introduces some important features that distinguish it from other numerical methods, such as the easy modeling of interactions among the fluids and full parallelism. During the past two decades, the LBM has attracted much attention and interest. There has been rapid progress in developing new models and applications in a multitude of fields, e.g. [2–5]. Although LBM has been successfully applied to simulate the isothermal flow problems, its application in the heat transfer system has not achieved such great success because of the severe numerical instability for the thermal models.

Generally, the current thermal models fall into the following categories: the multispeed method, and the multi-distribution function approach proposed by Nie, He, Chen, and Doolen [9–11]. For the multispeed method, two limitations severely restrict their applications, the narrow range of temperature variation and the severe numerical instability. The

* Corresponding authors.
E-mail addresses: ykun_wei@sina.com, yikunwei@zstu.edu.cn (Y. Wei), linzhe0122@zstu.edu.cn (Z. Lin).

https://doi.org/10.1016/j.amc.2018.07.047
0096-3003/

numerical stability and the range of temperature variation are improved in the multi-distribution function approach, which has been varied by several benchmark studies [12–14]. Although the multi-distribution function approach possess the obvious advantages, a slice of limitations still take on. In general, multi-distribution function method must be assumed that the Mach number of the flow is small and the density varies slowly to get the correct macroscopic equations from lattice Boltzmann equation [13]. So theoretically the multi-distribution function approach can only be used to simulate compressible flows in the incompressible limit. When the multi-distribution function approach is used for incompressible flows, it must be viewed as an artificial compressible method [16]. To reduce or eliminate such shortcomings, the extending lattice Boltzmann models were proposed for th Boussinesq incompressible flows [15–24]. Zou and Hou have proposed a modified equilibrium distribution and a modified velocity to construct an Lattice Boltzmann equation which models time-independent isothemal incompressible flows with significantly reduced compressibility [25], which significantly reduces the compressibility and is more robust.

Based on the above discussions, we mainly extend this incompressible model for thermal fluid flows by a modified equilibrium distribution function in this paper. A modified equilibrium distribution function is proposed in the present model for thermal dynamics equation. The present model is validated by the numerical simulation of the natural convection and Rayleigh – Bénard convection at a wide range of Rayleigh numbers, and Rayleigh – Taylor instability. Results of present model are in excellent agreement with that of previous LBM, DNS and theoretical prediction, which shows that the present model is also an efficient numerical method for natural convection, Rayleigh–Bénard convection, and Rayleigh–Taylor instability.

The rest of the paper is organized as follows: a novel lattice Boltzmann thermal model for incompressible flow will be detailedly described at first. After that, the detailed numerical study of the natural convection, Rayleigh –Benard convection and the two-dimensional Rayleigh–Taylor instability is presented and discussed in detail. The results are compared with previous LBM, the theoretical prediction and other computational results. A brief conclusion is given in final section.

2. Coupled lattice Boltzmann model

In this section, a novel coupled lattice Boltzmann model in two-dimensional space will be proposed in the following section. The approach can also be used to develop other models either in two space. The standard coupled lattice Boltzmann model bases on a square lattice. At first, a standard coupled lattice Boltzmann model for incompressible flows are briefly reviewed, and a novel coupled lattice Boltzmann model is then developed for incompressible fluid flows.

2.1. Standard coupled lattice Boltzmann model

The governing equations for the standard thermal energy distribution model are [18]:

$$f_i(x_\alpha + c_{i\alpha}\delta t, t+\delta t) - f_i(x_\alpha, t) = -\omega_1\left[f_i(x_\alpha, t) - f_i^{(eq)}(x_\alpha, t)\right] \tag{1}$$

$$g_i(x_\alpha + c_{i\alpha}\delta t, t+\delta t) - g_i(x_\alpha, t) = -\omega_2\left[g_i(x_\alpha, t) - g_i^{(eq)}(x_\alpha, t)\right] \tag{2}$$

where f_i is the density distribution function and g_i is the temperature distribution function, f_i^{eq} is the equilibrium function for the density distribution function, g_i^{eq} is the equilibrium function for the temperature distribution function, $c_{i\alpha}$ is the ith discretized velocity and ω is the relaxation parameter. For the D2Q9 model used in this study, $c_{i\alpha}$ is chosen as following respectively:

D2Q9:

$$c_{i\alpha} = \frac{\delta x}{\delta t}\begin{bmatrix} 0 & 1 & 0 & -1 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 1 & 0 & -1 & 1 & 1 & -1 & -1 \end{bmatrix}, \tag{3}$$

The equilibrium function for the density distribution function is given as [18]:

$$f_i^{eq}(x_\alpha, t) = w_i\rho\left\{1 + \frac{c_{i\alpha}u_\alpha}{c_s^2} + \frac{u_\alpha u_\beta}{2c_s^2}\left(\frac{c_{i\alpha}c_{i\beta}}{c_s^2} - \delta_{\alpha\beta}\right)\right\} \tag{4}$$

where c_s is the speed of sound and w_i is the weight coefficients. Parameters in D2Q9 is given as:

$$c_s = \frac{1}{\sqrt{3}}\frac{\delta x}{\delta t}, w_i = \begin{cases} \frac{4}{9} & i=0 \\ \frac{1}{9} & i = 1 \sim 4. \\ \frac{1}{36} & i = 5 \sim 8 \end{cases} \tag{5}$$

The relation between the relaxation parameter ω_1 and the kinematic viscosity ν is:

$$\nu = c_s^2\left(\frac{1}{\omega_1} - \frac{1}{2}\right)\delta t. \tag{6}$$