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The onset of convection in a magnetic nanofluid layer with variable gravity effects

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ABSTRACT

This paper deals with the study of the onset of convection in a thin magnetic nanofluid layer which is heated from below under the influence of an applied magnetic field and a variable gravity field. The model used in this study includes the effect of Brownian diffusion, thermophoresis, and magnetophoresis. We applied the Chebyshev pseudospectral method to determine the numerical solutions and discussed the results for Rigid–Rigid, Rigid–Free and Free–Free boundary conditions for water and ester based magnetic nanofluids. The effect of significant parameters affecting the instability of the system has been investigated at the onset of convection.

1. Introduction

Magnetic nanofluids (MNFs or ferrofluids) i.e. nanofluids exhibiting both magnetic and fluid properties, have various applications in thermal management systems as a coolant, in energy conservation systems as a heat transfer medium and in bioengineering for the hyperthermia treatment for cancer [1]. Convection in MNFs is an area that attracted the attention of many researchers over the decades. The primary study of ferrofluid convection in the presence of a uniform vertical magnetic field was conducted by Finlayson [2]. He considered Free-Free and Rigid-Rigid boundaries and discussed the results for both gravity and gravity free (in the absence of body force) environment. The author concluded that the presence of a temperature gradient and temperature dependent magnetization generates a spatial variation which in turn induces convection in a ferromagnetic fluid. Later this work was continued by [3–6] under various aspects. A nonlinear stability analysis for a horizontal layer of magnetized ferrofluid which is heated from below under the presence of a vertical magnetic field was studied by Sunil and Mahajan [7] by applying a generalized energy method. More related (nonlinear) studies include [8,9]. One of the characteristic features of MNF is that the flow of the fluid, heat transfer process and movement of magnetic nanoparticles can be controlled with the use of a magnetic field [10]. Some experimental studies reported that the orientation and strength of the magnetic field have a significant effect on the thermophysical properties of MNF. Schwab et al. [11] performed an experimental study in order to investigate the effect of a homogeneous vertical magnetic field on the Bénard convection in a layer of ferrofluid. They calculated the critical temperature difference by measuring the effected thermal conductivity and reported that the results obtained agree well with the theoretical predictions. Another experimental investigation to study the heat transfer features of ferrofluid in a tube under the presence of applied magnetic fields

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was organized by Lajvardi et al. [12]. The authors consider a water based ferrofluid and reported that the ferrofluid thermophysical properties, like specific heat capacity or thermal conductivity, plays an important role in the enhancement of heat transfer of ferrofluid under the presence of a magnetic field. Other experimental studies include [13–16].

This is well known that the gravity field of the Earth varies with height from its surfaces in many of the large-scale convection phenomena occurring in the atmosphere, the mantle of the earth or the ocean. When the gravity field varies with the height, then the fluid experience different buoyancy forces at different points. Thus the study of fluid convection with variable gravity turns important. The pioneer study in this area was performed by Pradhan and Samal [17]. The authors consider a fluid layer which is heated from above or below under the presence of a gravity field varies with height. Straughan [18] investigated the convective instability in a variable gravity field by applying the linear stability theory and nonlinear energy theory. The author reported that a change (increasing or decreasing) in the gravity in a specific direction has a significant effect on the stability of the system. In the famous book [19], Straughan analyzed the stability of a fluid layer which is heated from below under the effect of a variable gravity field. The author considers various gravity fields, and evaluated the effect of linear and nonlinear gravity variation at the onset of convection. The same problem in the case of porous medium and under the presence of an internal heat source was investigated by Rionero and Straughan [20]. For more related study, the reader may referred to Refs. [21–25].

A detailed study of convective transport in nanofluids was conducted by Buongiorno [26]. He reported that out of the seven slip mechanisms that can generate a relative velocity between the base fluid and nanoparticles, Brownian motion and thermophoresis are two important mechanisms in nanofluids. By considering the effect of these two slip mechanisms, the author developed a set of four equations namely the conservation of mass equation, momentum equation, nanoparticle mass equation and thermal energy equation. Following Buongiorno's model, Nield and Kuznetsov [27] investigated the onset of natural convection in a nanofluid layer by applying the linear stability theory. The authors consider three boundary conditions viz. Free–Free, Rigid–Rigid and Rigid–Free, and reported that the oscillatory stability is possible in the case when nanoparticles concentrate near the bottom of the layer. Later, the same authors (Nield and Kuznetsov [28]) developed a new revised model in order to rule out the possibility of oscillatory convection. In addition to these studies, the problem *convective instability in nanofluids* has been investigated by many other researchers (see Refs. [29–33]). The area "magnetic field effect on the onset of nanofluid convection" has attracted the attention of many researchers due to its significance in industry, biochemical engineering, chemical engineering, and various physical phenomenon related to astrophysics and geophysics [34]. Yadav et al. [35] applied the linear stability theory to investigate the effect of a vertical magnetic field on the onset of nanofluid convection in an electrically conducting layer of nanofluid. A linear stability analysis of the onset of convection in a horizontal nanofluid layer subject of a vertical magnetic field was made by Gupta et al. [34]. The onset of nanofluid convection subject to an applied magnetic field has been the subject of much more recent research, see Refs. [36–40]. The problem related to convection in MNFs was examined by Mahajan and Arora [41] under the framework of linear stability analysis. The model used in their study includes the effect of Brownian motion, thermophoresis and magnetophoresis. The same problem in the case of a porous medium was studied by [42]. Some other relevant studies include [43,44].

In all of the above-cited studies, the effect of variable gravity on the onset of MNF convection subject to an external uniform magnetic field has not been noticed yet. Earlier, Gupta et al. [34] investigated the effect of magnetic field on the onset of nanofluid convection. Here, we extended this study by considering a MNF layer subject to the variable gravity field. The boundary conditions are also revised and the new boundary conditions for the nanoparticle volume fraction are chosen in a way such that the nanoparticle flux is zero on the boundaries. Following the work of Buongiorno [26] and Shliomis and Smorodin [5], we considered a model that incorporates the effect of Brownian diffusion, thermophoresis, and magnetophoresis. The effects of important parameters like variable gravity coefficient, Lewis number, Langevin parameter, basic-density Rayleigh number, the width of MNF layer, modified particle-density increment are analyzed at the onset of convection.

2. Governing equations

An infinite horizontal MNF layer of thickness d is considered under the presence of a magnetic field $\boldsymbol{H} = H_0^{ext}\boldsymbol{k}$ which acts along the vertical axis from outside the layer. The fluid is assumed to be incompressible and occupy the layer $z \in [0, d]$. The temperature at lower and upper boundary are taken as T_h and T_c respectively [see Fig. 1]. The gravity field which depends on the vertical component z is acting in the negative z-direction.

The MNF is considered to be incompressible and so the continuity is

$$\nabla \cdot \boldsymbol{q} = 0, \tag{1}$$

here $\boldsymbol{q}$ is the MNF velocity.

Following [2,7,19,28], the momentum equation under the Boussinesq approximation is

$$\rho_f\left(\frac{\partial \boldsymbol{q}}{\partial t} + \boldsymbol{q} \cdot \nabla \boldsymbol{q}\right) = -\nabla p + \mu \nabla^2 \boldsymbol{q} + \mu_0(\boldsymbol{M} \cdot \nabla)\boldsymbol{H} - \rho g(z)\boldsymbol{k}, \tag{2}$$

where ρ_f is the MNF density, t is the time, p is the pressure, μ is the MNF viscosity, μ_0 is the magnetic permeability of vacuum, $\boldsymbol{M}$ is the magnetization, ρ is the overall density of the MNF and $g(z) = g(1 + \epsilon h(z))$ is the variable gravity