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Pricing of American options, using the Brennan–Schwartz algorithm based on finite elements

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ABSTRACT

A finite element method and implicit time steps are used to determine the price of an American option. The algorithm of Brennan and Schwartz is adapted to this situation and we prove convergence. Numerical tests confirm the theoretical result and lead to a smaller error for the same computational effort, compared to the finite difference method.

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1. Introduction

Using the basic results by Black–Scholes–Merton one can give explicit formulas for the value of European options. No such formula is available for American option. Thus numerical methods have to be used. Various methods have been proposed, e.g. [1,5,7,9,10,12–14].

Brennan and Schwartz [5] introduced a direct method to solve the problem for a put option and Jaillet et al. [10] gave a complete proof for the convergence of the algorithm. Ikonen and Toivanen [9] examine the case of a call option. All approaches use the method of finite differences (FD) to approximate the spatial derivatives. In this paper we verify that the method of finite element (FE) may be used. We spell out the necessary changes to the algorithm in [10] and proof that the method converges. With a few simulations we verify and illustrate the result. We find that the maximal approximation error for the FE approach is smaller than for the FD approach.

The paper is organized as follow. In Section 2, the problem is formulated in terms of variational inequalities. Section 3 introduces the discrete problem. In Section 4 the algorithm of Brennan–Schwartz algorithm is given and justification for the FE discretization. The numerical experiments are given in Section 5.

2. Variational inequalities

Consider the formulation based on the Black–Scholes model [4] as established by Bensoussan and Lions [3]. The American put option with continuous dividend yield solves the following parabolic inequality:

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$$\begin{aligned} \frac{\partial P}{\partial \tau} + \frac{\sigma^2}{2} S \frac{\partial^2 P}{\partial S^2} + (r-q) S \frac{\partial P}{\partial S} - rP &\le 0 \\ P &\ge \Psi \\ \left(\frac{\partial P}{\partial \tau} + \frac{\sigma^2}{2} S \frac{\partial^2 P}{\partial S^2} + (r-q) S \frac{\partial P}{\partial S} - rP \right)(P - \Psi) &= 0 \end{aligned} \tag{1}$$

for $\tau \in [0, T)$ and $S \in (0, \infty)$. The above is subject to the terminal condition

$$P(S, T) = \Psi(S) \text{ for } S \in (0, \infty)$$

where $\Psi(S) = (K-S)^+ = \max(K-S, 0)$. $K > 0$ is the strike price, $r > 0$ the interest rate, q the dividend yield and $\sigma > 0$ the volatility [8]. It is convenient to apply a change of variables.

$$\begin{aligned} u(x,t) &= P(Ke^x, T-\tau) \\ \psi(x) &= \Psi(Ke^x) = \max\{K - K\exp(x), 0\} \end{aligned}$$

This transformation leads to an initial boundary value problem with constant coefficients. The function $u(x, t)$ satisfies:

$$\begin{aligned} \dot{u} - \mathcal{A}u &\ge 0 \\ u &\ge \psi \\ (\dot{u} - \mathcal{A}u)(u - \psi) &= 0 \end{aligned} \tag{2}$$

for $t \in (0, T]$ and $x \in \mathbb{R}$, with the initial condition $u(x, 0) = \psi(x)$. Let $\mathcal{A}$ denote the second order differential operator

$$\mathcal{A} = \frac{\sigma^2}{2} \frac{\partial^2}{\partial x^2} + \mu \frac{\partial}{\partial x} - r \quad \text{where } \mu = r - q - \frac{\sigma^2}{2}$$

As spatial domain for (2) we use the finite domain $\Omega = [-L, +L]$ with the boundary conditions [7,10]:

$$u(-L, t) = \psi(-L) \text{ and } u(+L, t) = 0$$

Problem (2) can be transformed into a variational problem (see [1]):

$$\left(\frac{\partial u}{\partial t}, v - u \right) + a(u, v-u) \ge 0, \ \forall v \ge \psi \tag{3}$$

where $(\cdot, \cdot)$ is usual inner product on $L^2(\Omega)$ and $a(\cdot, \cdot)$ the bilinear form associated with the operator $\mathcal{A}$ given by

$$a(u, v) = \frac{\sigma^2}{2} \int_\Omega \frac{\partial u}{\partial x} \frac{\partial v}{\partial x} dx - \mu \int_\Omega \frac{\partial u}{\partial x} v \, dx + r \int_\Omega u v \, dx$$

and defined on $H^1(\Omega)$.

3. Discretization

To discretize the initial boundary value problem we use a simple, linear FE scheme for the spatial derivatives and an implicit scheme for the time discretization.

3.1. Time discretization

We discretize the time interval $[0, T]$ into subintervals of equal length $\Delta t = \frac{T}{N}$. This leads to $0 = t_0 < t_1 < \cdots < t_N = T$. For the time discretization we use a backwards difference approximation $\dot{u}(t) \approx \frac{1}{\Delta t}(u(t) - u(t - \Delta t))$. We denote the approximate solution at time $t_n = n\,\Delta t$ by u^n. For each time step we need to solve an elliptic variational inequality

$$\left(\frac{u^{n+1} - u^n}{\Delta t}, v - u^{n+1} \right) + a\left(u^{n+1}, v - u^{n+1}\right) \ge 0 \text{ for all } v \ge \psi \tag{4}$$

with the initial condition $u^0 = \psi$.

3.2. Space discretization

For the spatial discretization of (4) we use a finite element approach with piece-wise linear basis functions. For details see [14] or [7]. Divide the interval $[-L, +L]$ into $m+1$ subintervals $-L = x_0 < x_1 < \cdots < x_{m+1} = +L$ of equal length $h = \frac{2L}{m+1}$. Let $u_h^n = \left(u_1^n, \cdots, u_m^n\right)^T$ denote the approximating vector of the solution $u(x, t)$ at time $t = n\,\Delta t$ and position x. Now Eq. (4) reads as

$$\left(\frac{u_h^{n+1} - u_h^n}{\Delta t}, v_h - u_h^{n+1} \right) + a\left(u_h^{n+1}, v_h - u_h^{n+1}\right) \ge 0 \text{ for all } v_h \ge \psi \tag{5}$$

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