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An inverse time-dependent source problem for a time–space fractional diffusion equation☆

Y. S. Li[a,b], T. Wei[a,*]

[a] School of Mathematics and statistics, Lanzhou University, Gansu 730000, PR China
[b] School of Cyber Security, Gansu Political Science and Law Institute, Gansu 730000, PR China

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ABSTRACT

This paper is devoted to identify a time-dependent source term in a time–space fractional diffusion equation by using the usual initial and boundary data and an additional measurement data at an inner point. The existence and uniqueness of a weak solution for the corresponding direct problem with homogeneous Dirichlet boundary condition are proved. We provide the uniqueness and a stability estimate for the inverse time-dependent source problem. Based on the separation of variables, we transform the inverse source problem into a first kind Volterra integral equation with the source term as the unknown function and then show the ill-posedness of the problem. Further, we use a boundary element method combined with a generalized Tikhonov regularization to solve the Volterra integral equation of the fist kind. The generalized cross validation rule for the choice of regularization parameter is applied to obtain a stable numerical approximation to the time-dependent source term. Numerical experiments for six examples in one-dimensional and two-dimensional cases show that our proposed method is effective and stable.

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1. Introduction

The time–space fractional diffusion equation $\partial_{0+}^{\alpha} u(x,t) = -(-\Delta)^{\frac{\beta}{2}} u(x,t) + f(x,t)$ with $0 < \alpha < 1$ and $1 < \beta < 2$ is used to model anomalous diffusion [16]. The fractional derivative in time can be used to describe particle sticking and trapping phenomena and the fractional space derivative models long particle jumps. The combined effect produces a concentration profile with a sharper peak, and heavier tails. The direc t problems, i.e., initial value problems and initial boundary value problems for time–space fractional diffusion equations have attracted much more attention in recent years, for instances, Jia and Li [11] gave the maximum principles for the classical solution and weak solution. Chen et al. in [1] develops weak solutions of time–space fractional diffusion equations on bounded domains. Ding and Jiang in [5] consider the analytical solutions of multi-term time–space fractional advection-diffusion equations with mixed boundary conditions on a bounded domain.

However, in some practical situations, part of boundary data, or initial data, or diffusion coefficient, or source term may not be given and we want to find them by additional measurement data which will yield some fractional diffusion inverse problems. For the time fractional diffusion equations cases, the inverse source problems have been widely studied, On the

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* Corresponding author.
E-mail addresses: lys0311@163.com (Y. S. Li), tingwei@lzu.edu.cn (T. Wei).

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uniqueness of inverse problems, Cheng et al. in [2] gave the uniqueness results for determining the order of fractional derivative and space-dependent diffusion coefficient in a fractional diffusion equation by means of observation data on the boundary. Sakamoto and Yamamoto in [19] established a few uniqueness results for several inverse problems. Yamamoto and Zhang [28] provided a conditional stability for determining a zeroth-order coefficient in a half-order fractional diffusion equation. On numerical computations of inverse problems, Liu et al. [15] recovered a time-dependent factor in the unknown boundary condition for a time fractional diffusion equation by a nonlocal measurement condition. Yang et al. [29] solved an inverse problem for identifying the unknown source by the Landweber iterative regularization method. Wei and Wang in [26] solved an inverse space-dependent source problem by a modified quasi-boundary value method. Wei and Zhang in [27] proposed a numerical method to solve the inverse time-dependent source problem. Wei et al. in [25] identified a time-dependent source term in a multidimensional time-fractional diffusion equation from the boundary Cauchy data. As we know, the researches on inverse problems for time–space fractional diffusion equations are still lack of wide attention.

In this paper, we consider the following time–space fractional diffusion equation

$$\partial_{0+}^{\alpha} u(x,t) = -(-\Delta)^{\frac{\beta}{2}} u(x,t) + f(x)p(t), \qquad (x,t) \in \Omega_T, \tag{1.1}$$

where $\Omega_T := \Omega \times (0, T]$, $\Omega \subset R^d$ and $\alpha \in (0, 1)$, $\beta \in (1, 2)$ are fractional orders of the time and space derivatives, respectively, $T > 0$ is a fixed final time.

The fractional derivative ∂_{0+}^{α} denotes the Caputo fractional left-sided derivative of order α with respect to t defined by

$$\partial_{0+}^{\alpha} u(x,t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{\partial u(x,s)}{\partial s} \frac{ds}{(t-s)^{\alpha}}, \qquad 0 < \alpha < 1, \quad 0 < t \le T,$$

where Γ is the Gamma function .

The fractional Laplacian operator $(-\Delta)^{\frac{\beta}{2}}$ of order $\beta\,(1 < \beta \le 2)$ is defined by using the spectral decomposition of the Laplace operator. The definition is summarized in Definition 2.1 in Section 2. One can see Refs. [20–22].

Suppose the unknown function u satisfy the following initial and boundary conditions

$$u(x,0) = \phi(x), \qquad x \in \overline{\Omega}, \tag{1.2}$$

$$u(x,t) = 0, \qquad x \in \partial\Omega, \quad t \in (0,T]. \tag{1.3}$$

If all functions $f(x)$, $p(t)$, $\phi(x)$ are given appropriately, problems (1.1)–(1.3) is a direct problem. The inverse problem here is to determine the source term $p(t)$ in problem (1.1)–(1.3) from the additional data

$$u(x_0,t) = g(t), \qquad 0 < t \le T, \tag{1.4}$$

where $x_0 \in \Omega$ is an interior measurement location.

The inverse source problem mentioned above is an ill-posed problem, refer to Section 5. For the time–space fractional diffusion Eqs. (1.1)–(1.3), Tatar et al. in [20–22] solved an inverse space-dependent source problem and identified the orders of time and space fractional derivatives for a time–space fractional diffusion equation. Tuan and Long in [23] considered an inverse space-dependent problem to determine an unknown source term by Fourier truncation method, and give convergence estimates and regularization parameter choice rules, but no numerical example is provided. Kolokoltsov and Veretennikova in [13] studied the Cauchy problem for non-linear in time and space pseudo-differential equations. Dou and Hon in [6] solving a backward time–space fractional diffusion problem based on a kernel-based approximation technique. In this paper, we focus on the numerical reconstruction for the source term $p(t)$ in (1.1)–(1.4). A generalized Tikhonov regularization method based on a boundary element method is used to determine the source $p(t)$.

The remainder of this paper is organized as follows. In Section 2, we present some preliminaries used in Sections 3–5. The existence and uniqueness of a weak solution for the direct problems (1.1)–(1.3) is proved in Section 3. In Section 4, we provide the uniqueness and a stability estimate for the inverse source problem. In Section 5, we propose a regularized method based on the boundary element discretization for recovering a stable approximation to $p(t)$. The numerical results for six examples in one-dimensional and two-dimensional cases are investigated in Section 6. Finally, we give a conclusion in Section 7.

2. Preliminary

Let $AC[0, T]$ be the space of functions f which are absolutely continuous on $[0, T]$. Throughout this paper, we use the following definitions and propositions given in [20–22] and [12].

Definition 2.1. Suppose $\{\bar{\lambda}_k, \varphi_k\}$ be the eigenvalues and corresponding eigenvectors of the Laplacian operator $-\Delta$ in Ω with Dirichlet boundary condition on $\partial\Omega$:

$$\begin{cases} -\Delta\varphi_k = \bar{\lambda}_k\varphi_k, & in \quad \Omega, \\ \varphi_k = 0, & on \quad \partial\Omega. \end{cases}$$

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