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An improved empirical mode decomposition method based on the cubic trigonometric B-spline interpolation algorithm

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ABSTRACT

Empirical mode decomposition (EMD) is a new method presented recently for analyzing nonlinear and non-stationary signals. Its basic idea is to decompose the signal into a series of complete orthogonal intrinsic mode functions (IMFs) based on the local characteristics of the signal in time domain. The key step of EMD is to use the cubic spline interpolation to connect the maximum and minimum values of the signals into upper and lower envelopes respectively, and then calculate the mean values of upper and lower envelopes. Based on the cubic trigonometric B-spline interpolation algorithm, a new improved method for EMD is proposed named CTB-EMD in this paper. In this method, the interpolation curve is more flexible because of the adjustability of shape of the cubic trigonometric B-splines curve. Thus, the overshoot and undershoot problems in the cubic spline interpolation curve can be avoided, and then the decomposition of the signal is more accurate and effect. Through numerical experiments, we compare the effect of this method with other methods on decomposing simulation signals and real signals. Experimental results show that this method can decompose signals more effectively and accurately.

1. Introduction

In the development of signal processing, Fourier transform and other methods such as fast Fourier transform and wavelet transform have been widely used by lots of scholars and technicians for long because of their simple and effective analysis for signals. However, because of using a global transformation, traditional Fourier methods cannot express the local time-frequency property of signals, which is the most fundamental and critical property for non-stationary signals. In other words, traditional Fourier methods are not suitable for the analysis of non-stationary data, and there are various reasons such as linearity, periodicity or stationarity provided to support it [5,23,24, 28–34]. Actually, in the complex electromagnetic environment of reality, the electromagnetic signals are usually on-stationary due to the existence of electromagnetic interference (EMI, or for short). Therefore, some of the methods mentioned above are no longer applicable [1,2]. If we use Fourier transform or wavelet analysis or other traditional methods to analyze EMI signals, there will inevitably lead to serious errors and even wrong conclusions [3,4,20,21,26,27].

For this, many researchers have put forward a lot of ways to improve this situation. Recently it has been established that Fourier method can be used for nonstationary time series or data analysis [22]. They provided the Fourier method termed

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the Fourier decomposition method (FDM), which generates a set of a small amount of band limited Fourier intrinsic band functions (FIBFs). It makes the Fourier theory improve to a higher degree. Earlier, Huang et al. proposed a new method for analyzing nonlinear and non-stationary signals in 1998: Empirical Mode Decomposition (EMD) [5]. This method cannot only deal with linear and stationary signals, but also deal with nonlinear and non-stationary signals. Since the frequency range of noise and useful components in EMI signals is different, the signal can be decomposed into different frequency basis functions to obtain clean signals. The basic idea of the method is to decompose the signal into a set of complete orthogonal intrinsic mode functions (IMFs) based on the local characteristics of the signal in time domain. And these finite IMFs will produce the instantaneous frequency with practical significance in terms of time through the Hilbert transform. In this way, the signal can be analyzed effectively both in time domain and frequency domain, which is not available in the previous signal analysis methods based on Fourier transform. Through the analysis of the EMD algorithm, we can see that Fourier transform is actually a special case of EMD, that is, when the amplitude and frequency are both constant. Because of the effectiveness of EMD method in dealing with non-stationary signals, it has been widely used in various fields since it was put forward: signal denoising, mechanical error diagnosis, image processing, speech signal processing, seismic signal processing, crack detection and supervision, etc. Satisfactory results were also obtained for its processing results [6–8]. But it still has some unavoidable drawbacks such as the mode mixing effect caused by intermittent signals or noises. This will make the physical meaning of individual IMF not clear or even distort [9]. In order to solve this problem, Huang et al. proposed a noise-assisted data analysis method called ensemble empirical mode decomposition (EEMD) [10].The main idea is to add a Gauss white noise to the original signal. Because the frequency information of white noise is more abundant, it can fill the blank frequency domain of the original signal to make the sifting result better and alleviate the intermittent problem of the original signal frequency. On this basis, some scholars have put forward a series of improvement methods, including the complete ensemble empirical mode decomposition (CEEMD) [11], the complete ensemble empirical mode decomposition with adaptive noise (CEEMDAN) [12], and so on.

These methods are improved on the whole algorithm of EMD, which are equivalent to do a preprocessing for the signal, and achieved good results. In addition, there is some improvement aimed at the EMD algorithm itself, which improves the interpolation method during the EMD iteration processing, such as B-spline interpolation based EMD(B-EMD) [13,19,25] and cardinal spline interpolation based EEMD(C-EEMD) [14]. In addition, to reduce mode mixing and detrend uncertainty, [23,24] proposed a method which uses non-polynomial cubic spline interpolation to obtain upper and lower envelopes which improves orthogonality among IMFs.

This article aims at the EMD method itself and modifies the interpolation method of the envelope which is constructed based on the cubic trigonometric B-spline interpolation method. The cubic spline curve is widely used in engineering because of its simple structure and flexibility. However, the interpolation algorithm has its disadvantage, that is, the shape of the interpolation is uncontrollable, leading to overshoot and undershoot problems. In contrast to the cubic spline interpolation curve, the cubic trigonometric B-spline method introduces a shape parameter λ in the basis function, which will increase the flexibility of shape adjustment so as to increase the degree of freedom to avoid the overshoot and undershoot problems existing in the cubic spline interpolation curve effectively. The curve constructed by it not only has the local controllability and auto smoothness of the cubic spline curve, but also has the shape adjustability and the higher order of continuity. If the parameters are introduced, the spline basis functions need to be reconstructed and implemented on the algorithm, and then we can apply it to the EMD algorithm. Our approach is to use it to replace the cubic spline interpolation function in the EMD algorithm.

The main innovations of this paper are as follows: 1. A novel cubic trigonometric B-spline interpolation algorithm is proposed and implemented in program; 2. Apply it to EMD algorithm and produce a new EMD algorithm based on cubic trigonometric B-spline interpolation and verify the decomposition effect; 3. Decomposed the actual EEG signal by this method. The framework of this paper is as follows: The cubic trigonometric B-spline interpolation algorithm is introduced in the second part; The third part briefly introduces the EMD algorithm and the EMD algorithm based on the cubic trigonometric B-spline interpolation; Finally, numerical experiments are carried out and compared with other methods.

2. Cubic trigonometric B-spline interpolation algorithm

2.1. Preliminary knowledge

Because the cubic Bézier curves and cubic B-spline curves have the advantages of simple structure and high flexibility, they are used most widely in engineering practice and also discussed most in papers. [15] gave a set of cubic trigonometric Bézier (T-Bézier) basis functions with two parameters λ and μ; [16] proved that when $\lambda,\ \mu \in (-2, 1]$, the basis function in [15] is the optimal normalized totally positive basis in the corresponding space. Therefore, the cubic T-Bézier curves defined by it have the convexity preserving and the variation diminishing properties. And thus, the shape of the control polygon can be best simulated. But this curve belongs to the type of Bézier, and therefore do not have local control property and automatic smoothness. Now we construct a set of normalized totally positive basis based on this optimal normalized totally positive basis and define a piecewise composite curve whose structure is the same as the cubic B-spline curve. The curve has the local controllability and automatic smoothness with the same as the B-spline method. Besides, it also has the shape adjustability and the higher order of continuity.