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The nonlinear analysis for a new continuum model considering anticipation and traffic jerk effect

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ABSTRACT

Considering the anticipation and traffic jerk simultaneously, a new macro model of traffic flow is put forwarded in this paper. By means of linear stability theory, the new model's linear stability with the effect of traffic jerk and anticipation term is obtained.

Through nonlinear analysis, the KdV–Burgers equation is derived to describe the propagating behavior of traffic density wave near the neutral stability line. Numerical simulation is carried out to study the influence about the traffic jerk and anticipation effect. The evolution of traffic jam and corresponding energy consumption are then explored and compared each other. The outputs of these findings demonstrate the extended model can suppress traffic jam and reduce energy consumption in real traffic condition.

1. Introduction

In recent years, traffic congestion has an important influence on modern society since traffic problem become more and more serious with the increases of traffic flux. The traffic jam has closely impacted on human's daily life, such as traffic accidents, air pollution and global warming [1–13]. Therefore, many traffic models have been presented, such as the car-following models [14–26], the cellular automaton models [27–30], the gas kinetic models [31–33], and the hydrodynamic models [34–36], and to figure out the complicated constitution behind the traffic congestion phenomenon.

In 1995, Bando et al. [37] proposed the optimal velocity model (OVM), which can be used to explain the qualitative characteristics of the actual traffic flow, such as the stop-and-go phenomenon, traffic instability and the congestion evolution and so on. Based on the OVM, many new car-following models have been presented [14–26]. However, the OVM has the drawbacks of high acceleration rate and unrealistic deceleration. To improve OVM, Jiang et al. [38] put forward a full velocity difference model (FVDM) with the consideration of full velocity difference.

In many traffic situations, the sudden braking and acceleration can lead to a large waste of energy, in addition, it is harmful to the vehicles and it will increase the environmental pollution [39–41]. The temporal dynamics of the acceleration and deceleration of a vehicle is represented by the jerk profile. As we know, jerk is the derivative of the acceleration. In real traffic, drivers often adjust his speed according his anticipation by observing the traffic condition [42–46]. Until now, the investigation of incorporating of driver's jerk and anticipation on the continuum models is rare. In fact, traffic jerk and

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driver's anticipation will have a significant impact on traffic movement. In view of this, a new continuum model will be presented to investigate these influences on traffic flow.

In this paper, a new macro traffic flow model by incorporating the effect of driver's anticipation and traffic jerk based on OVM is presented. The linear and non-linear analysis for this new traffic model is obtained by means of linear and non-linear theory. The KdV–Burgers equation is derived and the soliton solution is given. Theoretic analysis and numerical simulation has been proposed to explore this complex phenomenon resulted. Additionally, the energy consumption is a recently concerned problem and we also attempt to discover energy consumption of proposed model.

2. The new continuum model

Taking both the driver's anticipation and traffic jerk term into the OVM, a new dynamic equation is obtained as follows:

$$\begin{cases} \dfrac{dv_n(t)}{dt} = a[V(\Delta x_n(t) + \tau \Delta v_n) - v_n(t)] - \lambda J_n(t), \\ J_n(t) = \dfrac{dv_n(t)}{dt} - \dfrac{dv_n(t-\sigma)}{dt}, \end{cases} \tag{1}$$

where $\Delta v_n = v_{n+1} - v_n$ is the velocity difference between the preceding vehicle $n+1$ and the following vehicle n, τ represent the anticipation time, a is the sensitivity of a driver, λ is the jerk parameter, σ is the memory time. Eq. (1) shows that the acceleration of the nth vehicle at time t is determined not only by the velocity $v_n(t)$, but also by the anticipation time and the traffic jerk.

Before deducing the continuum model, we make first-order Taylor expansion of the micro variables $V(\Delta x_n(t) + \tau \Delta v_n(t))$ and neglect high-order terms, i.e.

$$V(\Delta x_n(t) + \tau \Delta v_n(t)) = V(\Delta x_n(t)) + \tau \Delta v_n(t) V'(\Delta x_n(t)). \tag{2}$$

Make the Taylor series expansion of $v(x+\Delta, t)$ to the second-order term and neglect the high-order terms, we will obtain

$$v(x+\Delta, t) = v(x,t) + v'(x,t)\Delta + \frac{1}{2} v''(x,t)\Delta^2. \tag{3}$$

The above micro variables can be transformed into the macro ones as follows:

$$\begin{array}{lll} v_n(t) \to v(x,t), & v_{n+1}(t) \to v(x+\Delta, t), & \\ V(\Delta x_n(t)) \to V_e(\rho), & V'(\Delta x_n(t)) \to \bar{V}'(h), & J_n(t) \to v(x,t) v_{xt}(x,t), \end{array} \tag{4}$$

where Δ represents the distance between two adjacent vehicles, $\rho(x, t)$ and $v(x, t)$ represents the macro density and speed at place (x, t). Through the density ρ and the mean headway $h = \frac{1}{\rho}$, we define the equilibrium speed $V_e(\rho)$ and have $\bar{V}'(h) = -\rho^2 V_e'(\rho)$. Putting the above macro variables into Eq. (1), the following equation is derived

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = \frac{dv(x,t)}{dt} = a[V_e(\rho) - v] - a\tau \rho^2 V_e'(\rho) v'(x)\Delta - \lambda \sigma v v_{xt}. \tag{5}$$

Eq. (5) can be rewritten as

$$\frac{\partial v}{\partial t} + (v + a\tau\rho^2 V_e'(\rho)\Delta)\frac{\partial v}{\partial x} = a[V_e(\rho) - v] - \lambda\sigma v v_{xt}. \tag{6}$$

A new macro model considering traffic jerk and anticipation effect is derived by incorporating the conservative equation with Eq. (6):

$$\begin{cases} \dfrac{\partial \rho}{\partial t} + \dfrac{\partial(\rho v)}{\partial x} = 0, \\ \dfrac{\partial v}{\partial t} + (v + a\tau\rho^2 V'_e(\rho)\Delta)\dfrac{\partial v}{\partial x} = a[V_e(\rho) - v] - \lambda\sigma v v_{xt}. \end{cases} \tag{7}$$

3. Stability analysis

For the convenience of analysis, Eq. (7) is expressed in the vector form

$$\frac{\partial \mathbf{U}}{\partial t} + \mathbf{A}\frac{\partial \mathbf{U}}{\partial x} = \mathbf{E}, \tag{8}$$

where

$$\mathbf{U} = \begin{bmatrix} \rho \\ v \end{bmatrix}, \tag{9}$$

$$\mathbf{A} = \begin{bmatrix} v & \rho \\ 0 & v + a\tau\rho^2 V'_e(\rho) \end{bmatrix}, \tag{10}$$