Applied Mathematics and Computation 328 (2018) 162–170

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Replicator dynamics for public goods game with resource allocation in large populations

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ARTICLE INFO

Keywords:
Peer punishment
Cooperation
Reward
Resource allocation

ABSTRACT

Costly punishment can promote human cooperation, but the effectiveness of punishment is reduced because of the existence of second-order free-rider problem. How to solve the problem remains a challenge for the emergence of costly punishment. Motivated by the regimes of resource allocation in human society, in this work we consider the resource allocation with threshold for the common pool in the public goods game with an additional strategy of peer punishment, and aim to explore whether such proposed resource allocation can solve the problem of second-order free-riders by using replicator equations in infinite well-mixed populations. We assume that if contributing resources in the common pool exceed the threshold, the contributing resources will be divided into two parts: the first part will be equally allocated by all the players, and the second part will be allocated by all the players based on their strategy choices. Otherwise all the contributing resources are equally allocated by all the players. We find that the second-order free-rider problem can be effectively solved by this regime of resource allocation even when most of contributing resources are equally allocated among individuals. In addition, we find that punishment is the dominant strategy in a broad region of allocation parameters. Our work may thus suggest an effective approach about resource allocation for resisting second-order free-riders in the public goods dilemma.

1. Introduction

Cooperative phenomenon is pervasive in nature and human society [1–3]. However, the emergence of cooperation is still an unsettled problem across behavioral disciplines ranging from microbial populations to human society. Evolutionary game theory provides an important and effective theoretical framework for studying the evolution of cooperation [4–10]. In particular, the public goods game (PGG), which is a standard metaphor of social dilemmas, has aroused extensive attention of social scientists, economists, and biologists [11–14].

In the PGG, cooperators confer benefits on others with some costs to themselves, whereas defectors contribute nothing and exploit the benefits. Therefore, from the viewpoint of individual selfishness, natural selection will favor defective behavior and drive elimination of cooperative behavior. Correspondingly, the tragedy of the commons will be caused [15]. In order to resist this tragedy, many mechanisms have been verified to promote the evolution of cooperative behavior, such as direct reciprocity [16], reputation [17,18], group selection [19,20], punishment [21–24], exclusion [25], reward [26–28], spatial reciprocity [29–33], and so on. In particular, punishment, as a feasible mean for promoting the evolution of cooperation, has

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https://doi.org/10.1016/j.amc.2018.01.045
0096-3003/

been widely studied in recent years [34–39]. However, since punishment is costly, cooperators may refuse to pay a cost to punish defectors, which ultimately incurs the second-order free-rider problem. Therefore, how to promote the emergence of costly punishment remains a challenge [40–42]. In order to solve this problem, most of works are mainly paying attention to punish second-order free-riders based on the traditional PGG [25,28,43,44]. However, this way may reduce the payoff of punishment and be not conducive to the emergence of punishment [45].

On the other hand, providing reward for altruists is often used in human society for promoting the emergence of punishment behavior and can increase the earnings of punishers [28]. For example, in an enterprise in addition to the basic wage an employee will be provided with an extra bonus as an important part of his/her salary if he/she does an excellent working performance for the enterprise. Otherwise, he/she can only have the basic wage for the salary. And the bonus is often allocated according to the degree of his/her contribution to the enterprise, and indeed it can be viewed as a kind of reward for the altruists to the whole community. And this allocation regime for the source of salary can be often found in human society, but in theory it is still unclear how such allocation regime can affect the evolution of cooperation and whether it can solve the second-order free-rider problem.

Inspired by the above consideration, in this work we propose an allocation regime for the common pool of the public goods game with peer punishment strategy, and aim to study how such allocation regime influences the evolutionary dynamics of cooperators, defectors, and punishers in a large well-mixed population. In particular, we will focus on whether such allocation regime can effectively solve the second-order free-rider problem. We assume that if contributing resources in the common pool exceed a fixed threshold, the contributing resources will be divided into two parts: the first part will be equally allocated by all the players, and the second part will be rewarded by all the players based on their strategy choices. Otherwise all the contributing resources are equally allocated by all the players, as done in the traditional PGG [22,25,26]. We emphasize that in our model when the contributing resources exceed the threshold, each player can be rewarded no matter what strategy they choose because of its participation into the game, and meanwhile the rewarded amount is determined based on their contribution degree to the common pool. Considering this frame of resource allocation, we study evolutionary dynamics of three strategies by means of replicator equations, and we find that the second-order free-rider problem can be effectively solved by this regime of resource allocation. In addition, we demonstrate that punishment is the dominant strategy in a broad region of allocation parameters.

2. Model and method

We consider the PGG with peer punishment in an infinitely large, well-mixed population. From time to time, G individuals of population are randomly chosen and formed a group. In the game, each cooperator contributes c to the common pool, whereas defectors contribute nothing. Each punisher also contributes c to the common pool, and imposes a fine α on each defector, at a cost β. Subsequently, the total contributions of the group are multiplied by the synergy factor r $(1<r<G)$, which represents the contributing resources. Here the amount of the contributing resources is $(G-G_D)rc$, where G_D is the number of defectors among the G individuals. If contributing resources in the public goods pool are not more than resource threshold λGrc $(0\le\lambda\le1)$, all the contributing resources are equally divided by all the players. Otherwise the contributing resources will be divided into two parts: the first part λGrc will be equally allocated by all the players, and the second part $(G-G_D)rc-\lambda Grc$ will be used to reward all the players based on the certain reward weights according to their strategy choices. Here, the second part is the rewarding resources. We emphasize that the parameter λ represents the strength of resource threshold. When $\lambda=1$, it means that the total contributing resources will be equally allocated among all the participants, which recovers to the traditional PGG model. While when $\lambda=0$, it means that the total contributing resources will be allocated among all the participants according to their strategy choices.

Based on the above description, accordingly the payoffs Π_i of the three strategies $(i=C,D,P)$ in the group can be respectively given as

$$\Pi_C=\sigma(r(G-N_D)c-\lambda Grc)\left[\lambda rc+\frac{a(r(G-N_D)c-\lambda Grc)}{a(N_C+1)+bN_D+N_P}\right]+[1-\sigma(r(G-N_D)c-\lambda Grc)]\frac{r(G-N_D)c}{G}-c, \tag{1}$$

$$\Pi_D=\sigma(r(G-1-N_D)c-\lambda Grc)\left[\lambda rc+\frac{b(r(G-1-N_D)c-\lambda Grc)}{aN_C+b(N_D+1)+N_P}\right]+[1-\sigma(r(G-1-N_D)c-\lambda Grc)]\frac{r(G-1-N_D)c}{G}-\alpha N_P, \tag{2}$$

and

$$\Pi_P=\sigma(r(G-N_D)c-\lambda Grc)\left[\lambda rc+\frac{(r(G-N_D)c-\lambda Grc)}{aN_C+bN_D+N_P+1}\right]+[1-\sigma(r(G-N_D)c-\lambda Grc)]\frac{r(G-N_D)c}{G}-c-\beta N_D, \tag{3}$$