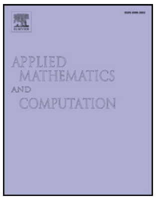
Applied Mathematics and Computation 326 (2018) 159–169

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc



Modelling of a two-phase vortex-ring flow using an analytical solution for the carrier phase

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ARTICLE INFO

MSC:
76T10

Keywords:
Vortex ring
Dusty gas
Two-phase flow
Fully Lagrangian approach

ABSTRACT

A transient axially symmetric two-phase vortex-ring flow is investigated using the one-way coupled, two-fluid approach. The carrier phase parameters are calculated using the approximate analytical solution suggested by Kaplanski and Rudi (Phys. Fluids vol. 17 (2005) 087101–087107). Due to the vortical nature of the flow, the mixing of inertial admixture can be accompanied by crossing particle trajectories. The admixture parameters are calculated using the Fully Lagrangian Approach (FLA). According to FLA, all of the dispersed phase parameters, including the particle/droplet concentration, are calculated from the solution to the system of ordinary differential equations along chosen particle trajectories; FLA provides high-accuracy particle number calculations even in the case of crossing particle trajectories (multi-valued fields of the dispersed media). Two flow regimes corresponding to two different initial conditions are investigated: (i) injection of a two-phase jet; and (ii) propagation of a vortex ring through a cloud of particles. It was shown that the dispersed media may form folds and caustics in these flows. In both cases, the ranges of governing parameters leading to the formation of mushroom-like clouds of particles are identified. The caps of the mushrooms contain caustics or edges of folds of the dispersed media, which correspond to particle accumulation zones.

1. Introduction

Two-phase vortex-ring flows are widely observed in engineering and environmental conditions [1,2], including direct injection internal combustion engines [3]. In such flows, the admixture forms high concentration regions with folds (local zones of crossing particle/droplet, hereafter referred to as particles, trajectories) and caustics. The Eulerian approaches cannot describe such regions with reasonable accuracy, since these approaches are based on the assumption of single-valued fields of the particle concentration and velocities. Ferry and Balachandar [4] showed that the condition for uniqueness of the particle velocity field is related to the particle response time and the maximal compressional strain of the dispersed phase flow. This uniqueness was shown to be expected for small compressional strains, and short particle response times, which correspond to small particle Stokes numbers. As shown by Healy and Young [5], the only method capable of calculating

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https://doi.org/10.1016/j.amc.2017.12.044
0096-3003/

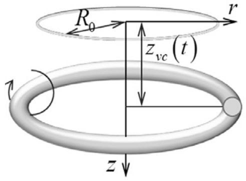

Fig. 1. Flow diagram of the vortex ring.

the particle concentration field in the case of multi-valued admixture parameter fields, without using excessive computer power, is the one suggested by Osiptsov [6], known as the Fully Lagrangian Approach (FLA). At the edge of a local region of crossing particle trajectories (caustics), the particle number density has a singularity. This is a well-known feature of the mathematical model of the collisionless continuum of point particles (see details in [7]). In the latter paper, typical examples of flows with singularities in the particle number density field were analysed. It was shown that for an integrable singularity of particle number density, at the singular points the mean distance between the particles remains finite and the model of collisionless particles remains valid. Our study is focused on further investigation of these types of flows based on the previously developed vortex ring models for the carrier phase and the Fully Lagrangian Approach (FLA) for the dispersed phase. Particular attention is paid to the details of the mixing process with regions of high particle concentration, which can potentially lead to the formation of unfavourable zones of high fuel vapour concentration in internal combustion engines, when particles are identified with fuel droplets.

Vortex-ring flows have been extensively studied theoretically and experimentally [1,2,8–14]. Theoretical studies have been mainly focused on vortex rings in the limits of high [10,11] and low [15,16] Reynolds numbers based on the initial velocity circulation and the ring radius (see [17] for an overview of vortex ring propagation models). In [18], the analytical solution suggested by Kaplanski and Rudi [19] was applied to the analysis of particle dynamics and mixing in an oscillating vortex pair, using the conventional Lagrangian approach. An alternative approach to simulation of the dynamics of particles in a 2D vortex pair formed by a plane jet, using the Fully Lagrangian Approach (but not the Kaplanski–Rudi solution) is described in [20].

Investigation of particle-laden vortex-ring flows is also important in studies of inertial-particle accumulation in turbulent flows. Yang and Shy [21] investigated particle distribution in turbulent flows experimentally, while Squires and Eaton [22], Goto and Vassilicos [23], Chen et al. [24], and Soldati and Marchioli [25] addressed it theoretically. These studies show that admixture distribution in turbulent flows crucially depends on the vortex structures. In the cases where the density of the particle material is higher than the density of the carrier phase, accumulation zones were shown to be formed on the edges of intense local vortices. These findings agree with the results of experimental and theoretical studies of the interactions of particles with vortex structures [26–28]. Foster et al. [27] studied particle concentrations in the framework of the Eulerian approach. Soldati and Marchioli [25] used Lagrangian tracking together with statistical tools in order to quantify particle segregation. Lebedeva and Osiptsov [28] used FLA to calculate particle number density fields in a steady-state, axially symmetric, tornado-like flow.

In the present study, a two-phase vortex-ring flow is considered in the framework of the two-fluid (inter-penetrating continua) approach [29]. According to this approach, a cloud of particles is described using continuum values of velocity and number density of the dispersed media. In contrast to previous studies, the approach used in our paper is based on the Kaplanski–Rudi [19] analytical solution for the carrier phase and the Fully Lagrangian Approach for the dispersed phase. The choice of such a combined (analytical and numerical) approach is supported by the fact that in the flow considered, the mixing process can be accompanied by crossing particle trajectories, the onset of local zones of multi-valued particle parameters, caustics and particle accumulation regions. The analytical solution for the carrier phase and the use of FLA for the dispersed phase allows us to calculate particle number density correctly with high accuracy. Our analysis is focused on an axially symmetric vortex ring rather than a plain vortex pair flow considered earlier in [28]. The Kaplanski–Rudi [19] analytical model was compared against DNS simulations in [13]. Analytical and numerical models showed good agreement. However, it is not clear if the combination of the analytical solution based on the self-similar variables with FLA could be applicable to simulate two-phase vortex ring flows.

The mathematical model of the two-phase vortex ring flow, used in the analysis, and mathematical formalism of the Kaplanski–Rudi model are discussed in Section 2. In Section 3, two-phase jet injection and the propagation of a vortex ring through a cloud of dust are considered. The main results of the paper are summarised in Section 4.

2. Formulation of the problem

We consider an axially symmetric transient flow of a two-phase gas-particle mixture, interacting with a vortex ring, and introduce a cylindrical coordinate system as shown in Fig. 1. The vortex ring propagates along the z-axis (the axis of symmetry); at the initial instant of time the vortex ring is assumed to be confined by the $z = 0$ plane, R_0 is the initial vortex ring radius.