Applied Mathematics and Computation 321 (2018) 244–254

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Optimal control for doubly nonlinear evolutionary inclusions

Leszek Gasiński [a], Stanisław Migórski [a], Anna Ochal [a], Zijia Peng [b,*]

[a] Faculty of Mathematics and Computer Science, Jagiellonian University in Krakow, ul. Lojasiewicza 6, Krakow 30348, Poland
[b] Guangxi Key Laboratory of Universities Optimization Control and Engineering Calculation, and College of Sciences, Guangxi University for Nationalities, Nanning, Guangxi 530006, PR China

ARTICLE INFO

MSC:
49J20
49J24
47J20

Keywords:
Hemivariational inequality
Evolutionary inclusion
Optimal control
Doubly nonlinear
Pseudomonotone operator

ABSTRACT

We study the optimal control of systems for a class of nonlinear hemivariational inequalities which are in the form of evolutionary inclusions involving Clarke's generalized gradient. The control variables are introduced both in the generalized gradient and in the source terms. We first establish the existence of weak solutions to nonlinear inclusions and prove the upper semicontinuity property of their solution sets. Then, we present the minimization problem and show the existence of optimal admissible state-control pairs. Finally, some examples of our abstract results which appear in applications are discussed.

1. Introduction

In this paper we consider optimal control problems of systems described by an evolutionary hemivariational inequality. Such an inequality is formulated as the evolutionary inclusion of first order with the multivalued Clarke subdifferential term. The inclusion is considered in the framework of evolution triple of spaces and the model involves two control variables, one is a distributed parameter control and the second one appears in the multivalued subdifferential term.

Our main results concern the upper semicontinuity of the solution set to the problem and the existence of optimal admissible state-control pairs for the optimal control problem with Lagrangian cost functional.

There has been extensive study on optimal control of evolution problems. For instance, the existence and approximation of optimal solutions and the necessary optimality conditions have been studied by Lions [1], Tröltzsch [2] for differential equations, and by Barbu [3] and Tiba [4] for variational inequalities. Also, the optimal control problems governed by differential inclusions of subdifferential type were studied by Tolstonogov [5]. Recently, the existence and convergence of the optimal control for quasi-variational inequalities have been studied by Khana and Samab [6], whereas the optimal control of nonlinear rate-independent evolution process has been investigated by Rindler [7]. As concerns control problems for hemivariational inequalities, they have been treated in the stationary case by Panagiotopoulos [8], Miettinen and Haslinger [9], Haslinger and Panagiotopoulos [10,11], Denkowski and Migórski [12]. As for the evolutionary case, these problems have been studied by Migórski and Ochal [13], and Park and Jeong [14]. Moreover, their application to piezoelectric frictional contact models has been considered recently by Denkowski, Migórski and Ochal [15]. However, optimal control problems for doubly nonlinear inclusions with nonmonotone perturbations, cf. (1) below, have not been considered in the literature. In the present paper we exploit the very recent results in [16–18] on the existence of solutions to hemivariational inequality with doubly nonlinear operators.

* Corresponding author.
E-mail address: pengzijia@126.com (Z. Peng).

https://doi.org/10.1016/j.amc.2017.10.041

The paper is organized as follows. First, in Section 2 we provide the motivation for our study. Section 3 is concerned with the preliminaries and hypotheses to hemivariational inequality problems. Then, in Section 4 we establish the existence results for these problems and discuss the properties of their solution sets. On the basis of Section 4, in Section 5 we study the optimal control problem and show the existence of optimal admissible state-control pairs. Finally, in Section 6, some examples of our abstract results are provided.

2. Motivation

The motivation for our study comes from the nonconvex superpotential problems in Mechanics and Engineering Sciences. Their variational formulations are hemivariational inequalities which were introduced by P.D. Panagiotopoulos in 1981 and they express the principle of virtual work in inequality form. The hemivariational inequality considered in the paper can be formulated in the form of the following evolutionary inclusion:

$$\begin{cases} v'(t)+A(t,y(t))+g(t)=f(t)+C(t)u(t) \quad \text{a.e. } t\in(0,T),\\ v(x,t)\in\widehat{\beta}(x,y(x,t)) \quad \text{a.e. } (x,t)\in\Omega\times(0,T),\\ g(x,t)\in\widehat{g}(x,t,w(x,t),y(x,t)) \quad \text{a.e. } (x,t)\in\Omega\times(0,T),\\ v(0)=v_0, \end{cases} \tag{1}$$

where $\widehat{\beta}$ and $\widehat{g}$ represent multivalued lower order terms, and u and w denote the control variables. More precisely, $\widehat{\beta}$ is a maximal monotone operator induced by a nondecreasing function which has a jump at some point while $\widehat{g}$ denotes the Clarke generalized gradient of a locally Lipschitz function which describes the multivalued and nonmonotone relations in the interior of the domain Ω.

System (1) arises in various physical models for distributed parameter control problems with phase change. Here, our study is motivated by a two-phase Stefan problem which describes the heat transfer in metal melting and solidification process. The enthalpy formulation of this problem can be described by the equation

$$\frac{\partial e(y)}{\partial t}+\operatorname{div}q=f \quad \text{in } \Omega\times(0,T),$$

where $e(y)$ represents the enthalpy per unit volume, q is the heat flux vector, f describes the density of heat sources in Ω, and $y=y(x,t)$ denotes the temperature at point $x\in\Omega$ and time $t\in(0,T)$. From the generalized Fourier heat conduction law $q=-k(y)\nabla y$ in $\Omega\times(0,T)$, where $k(y)$ is the thermal conductivity depending on the temperature, it follows that:

$$\frac{\partial e(y)}{\partial t}-\operatorname{div}(k(y)\nabla y)=f \quad \text{in } \Omega\times(0,T). \tag{2}$$

Enthalpy formulation (2) is the energy conservation equation that treats the solid phase, liquid phase and the solid-liquid interphase as a whole. On the interphase, the latent heat of solidification is released (or absorbed for melting) so that the energy has a jump across it, which affects $e(y)$ being multivalued at the solidification temperature. Note that either in liquid or solid phase, since no phase change occurs, the model is described by the parabolic heat conduction equation

$$y'(t)-\operatorname{div}(k(y)\nabla y)=f \quad \text{in } \Omega\times(0,T). \tag{3}$$

In this paper, it is supposed that $f=f_1+f_2$, where f_2 is given and f_1 is a known function of the temperature in the form of

$$-f_1(x,t)\in\partial j(x,t,w(x,t),y(x,t)) \quad \text{a.e.} \quad (x,t)\in\Omega\times(0,T), \tag{4}$$

where $\partial j(x,t,\eta,\xi)$ denotes the generalized gradient of a locally Lipschitz function $j(x,t,\eta,\xi)$ with respect to its last variable. The multivalued function $\partial j(x,t,\eta,\cdot):\mathbb{R}\to 2^{\mathbb{R}}$ is generally nonmonotone and may include vertical jumps. In the physicist language, it means that the law is characterized by the generalized gradient of a nonsmooth potential $j(x,t,\eta,\cdot)$. For more information on the Clarke generalized gradient, we refer to [19].

The variational formulation of (2) with condition (4) leads to a hemivariational inequality whose corresponding control problem can be formulated as in (1). Note that the variational formulation of the simpler Eq. (3) with (4) leads to an optimal control problem for a parabolic hemivariational inequality. This problem has been studied by Migórski and Ochal in [13] and applied to nonmonotone and nonconvex interior semipermeability problems. Their monotone counterparts are described by variational inequalities, which have been studied by Duvaut and Lions in [20] with $j(x,t,\eta,\cdot)$ being a proper, lower semicontinuous and convex function, which implies that $\partial j(x,t,\eta,\cdot)$ is a maximal monotone mapping. Note also that the monotone and nonmonotone relations as in (4) could be also considered on the boundary of a domain which lead to boundary variational or boundary hemivariational inequality problems, respectively.

3. Notations and preliminaries

For a Banach space X, we denote by $\|\cdot\|_X$ the norm in X, by $\langle\cdot,\cdot\rangle_X$ the duality pairing between X and its dual X^* and by w-X the space X equipped with its weak topology. Let $\Gamma_0(X)$ stand for the set of proper, convex and lower semicontinuous