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Modeling of material damage using finite elements and time homogenization in case of finite strain

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ABSTRACT

This work is aimed at numerical simulations of high-cycle fatigue behavior of elastomeric materials. The main concern, however, is not the fatigue life itself, but the changes of mechanical properties prior to failure. This is of special interest in the case of elastomers. In such a case, large strains must be considered and so does, consequently, nonlinearity of constitutive equations as well. The usual approach to fatigue analysis consists of modal analysis and application of linear cumulation rule for damage. This is not generally applicable to large-strain regime nor to nonlinear material models. The most general approach to such problem is a full simulation of damage cumulation in time domain. Such a simulation, however, would be prohibitively expensive for a large number of loading cycles. As a remedy, the method of homogenization in time domain may be applied to the problem. The method has been applied to various material models already (viscoelasticity, viscoplasticity, damage). This article shows its applicability to the problem of damage cumulation under large strains and a significant improvement in computational times over full simulation.

1. Introduction

The use of continuum mechanics in the simulation of degradation processes is more computationally demanding than traditional approaches to fatigue analysis. However, it enables complex analysis including all kinds of nonlinearity and stress redistribution due to damage evolution.

In order to reduce the computation time, homogenization in time domain can be used. Successful application of the homogenization in time domain has already been reported for modeling of viscoelastic and viscoplastic materials [7,13], for thermo-viscoelastic composite [12], fatigue under two superimposed cyclic loads with different frequencies [1], fatigue damage of bone [5], crystal plasticity and fatigue under large strain conditions [9], or to high cycle fatigue of steel, including fatigue crack growth [3].

We investigate the application of time homogenization to a model of damage cumulation based on the Ogden hyperelastic model, which is suitable for the description of soft materials such as rubber or biological tissues. A brief summary of large strain theory is presented in Section 2 and a description of the concept of continuum damage mechanics is given in Section 3. The homogenized version of the damage cumulation model is derived in Section 4. The accuracy and performance

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of the homogenized model is illustrated using a numerical example presented in Section 5. The results are described in Section 6 and further discussed in Section 7.

2. Large strain kinematics and hyperelasticity

Basic prerequisites of the theory used in this article are given in this section. More detailed treatment can be found in many textbooks on solid mechanics, e.g. [6].

The motion of a deformable body can be described using a nonlinear map $\boldsymbol{\varphi}$ that maps the reference configuration Ω_0 to the current configuration Ω. The coordinates of a particle in the current (deformed) and reference (undeformed) configuration are denoted $\mathbf{x}$ and $\mathbf{X}$, respectively, so that the following relation holds at a particular time t: $\mathbf{x} = \boldsymbol{\varphi}(\mathbf{X}, t)$. The deformation gradient is defined as $\mathbf{F} = \nabla \boldsymbol{\varphi}(\mathbf{X}, t)$. The right Cauchy–Green deformation tensor can be obtained as $\mathbf{C} = \mathbf{F}^{\mathrm{T}} \mathbf{F}$ and its spectral representation

$$\mathbf{C} = \sum_{i=1}^{3} \lambda_i^2 \mathbf{N}_i \otimes \mathbf{N}_i \tag{1}$$

leads to the notion of principal stretches λ_i and principal directions $\mathbf{N}_i$.

A hyperelastic material is defined by a strain energy density function W. The constitutive equation is expressed in terms of work-conjugate stress and strain measures. In this article, the second Piola-Kirchhoff stress tensor $\mathbf{S}$ and the Green–Lagrange strain tensor $\mathbf{E}$ are used:

$$\mathbf{S} = \frac{\partial W}{\partial \mathbf{E}} \quad \text{or} \quad \mathbf{S} = \frac{\partial W}{\partial \mathbf{E}} - p\,\mathbf{C}^{-1}, \tag{2}$$

where $\mathbf{E} = \frac{1}{2}(\mathbf{C} - \mathbf{I})$ and p denotes the hydrostatic pressure in the case of incompressible material.

Many particular forms of the strain energy density function have been proposed. The Ogden hyperelastic model, which is used in this work, is expressed in terms of principal stretches [10]:

$$W = \sum_{i=1}^{N} \frac{\mu_i}{\alpha_i} \left(\lambda_1^{\alpha_i} + \lambda_2^{\alpha_i} + \lambda_3^{\alpha_i} - 3 \right), \tag{3}$$

where μ_i and α_i are the parameters of the model.

3. Damage model

In continuum damage mechanics (CDM), the damage of material is described using the damage variable. This quantity carries the information about macroscopic effects of microscopic cracks, which are formed as a consequence of mechanical loading; namely, a decrease in stiffness is observed after the material is damaged. The simplest case is the state of isotropic damage, which may be described using a scalar damage variable D. The damage variable can take values between 0 and 1, where 0 corresponds to an undamaged material and 1 to maximum damage, i.e. macroscopic crack initiation. The damage variable relates the effective principal values of the second Piola-Kirchhoff stress tensor $\tilde{S}_i$ to the principal stresses S_i by

$$\tilde{S}_i = \frac{S_i}{1 - D}. \tag{4}$$

The principal stresses S_i are the eigenvalues of the second Piola-Kirchhoff stress tensor $\mathbf{S}$.

Ayoub et al. [2] employed CDM to derive the following rule for evolution of the damage variable

$$\dot{D} = \left(\frac{S_{\mathrm{eq}}}{A\,(1 - D)} \right)^{a}, \tag{5}$$

where a and A are parameters of the damage evolution model and S_{eq} is equivalent stress defined as

$$S_{\mathrm{eq}} = \sum_{i=1}^{3} \frac{\tilde{S}_i}{\frac{\partial \tilde{S}_i}{\partial \lambda_i}} \frac{\partial W}{\partial \lambda_i}. \tag{6}$$

4. Homogenization of the damage model

Asymptotic series and time averaging are used to derive the homogenized version of the damage model. The approach is used frequently to perform homogenization both in spatial domain and in temporal domain.

Apart from the physical time $\bar{t}$, there are two time scales used: the slow time t and the fast time τ. These are related by

$$\bar{t} = t + T\,\tau, \quad \tau = \frac{t}{\xi}, \quad \xi \to 0, \tag{7}$$