Applied Mathematics and Computation 000 (2017) 1–17

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

A (non-)hydrostatic free-surface numerical model for two-layer flows

Jan Bohacek[a,*], Abdellah Kharicha[b], Andreas Ludwig[a], Menghuai Wu[b], Ebrahim Karimi-Sibaki[b], Armin Paar[c], Michael Brandner[c], Leonel Elizondo[c], Thomas Trickl[c]

[a] *Department of Metallurgy, Montanuniversitaet Leoben, Franz-Joseph Strasse 18, 8700 Leoben, Austria*
[b] *Christian Doppler Lab for Advanced Simulation of Solidification and Melting, Dept. of Metallurgy, Montanuniversitaet Leoben, Franz-Joseph Strasse 18, 8700 Leoben, Austria*
[c] *Eisenwerk Sulzau-Werfen, R.&E. Weinberger AG, A-5441 Tenneck, Austria*

ARTICLE INFO

Article history:
Available online xxx

Keywords:
numerical modeling
free-surface model
centrifugal casting
solidification
advection-diffusion equation
shallow water equations

ABSTRACT

A semi-implicit (non-)hydrostatic free-surface numerical model for two layer flows is derived from the Navier–Stokes equations by applying kinematic boundary conditions at moving interfaces and by decomposing the pressure into the hydrostatic and the hydrodynamic part. When the latter is ignored, the algorithm conveniently transforms into a solver for a hydrostatic flow. In addition, when the vertical grid spacing is larger than the layer depths, the algorithm naturally degenerates into a solver for the shallow water equations. In this paper, the presented numerical model is developed for the horizontal centrifugal casting, a metallurgical process, in which a liquid metal is poured into a horizontally rotating cylindrical mold. The centrifugal force pushes the liquid metal toward the mold wall resulting in a formation of a sleeve with a uniform thickness. The mold gradually extracts the sensible and the latent heat from the sleeve, which eventually becomes solid. Often a second layer of another material is introduced during the solidification of the first layer. The proposed free-surface model is therefore coupled with the heat advection-diffusion equation with a stiff latent heat source term representing the solidification. The numerical results show a good agreement with measurements of temperatures performed in the plant. A validation of the proposed model is also provided with the help of using other numerical techniques such as the approximate Riemann solver for the two layer shallow water equations and the volume of fluid method.

1. Introduction

Quite a few numerical models have been derived from the Navier–Stokes (N–S) equations to study free-surface flows. The main task of these models is to account for the interface separating fluid domains and being generally in motion. A typical fluid flow problem may involve one, two or more immiscible fluids. No matter the numerical method used, calculation steps can be summarized as: (a) set the boundary conditions at the interface; (b) advance the interface in time; (c) identify the position of the interface. According to [1,2], the most common numerical methods in this field are

* Corresponding author.
E-mail address: bohacek.jan@gmail.com (J. Bohacek).

http://dx.doi.org/10.1016/j.amc.2017.03.030

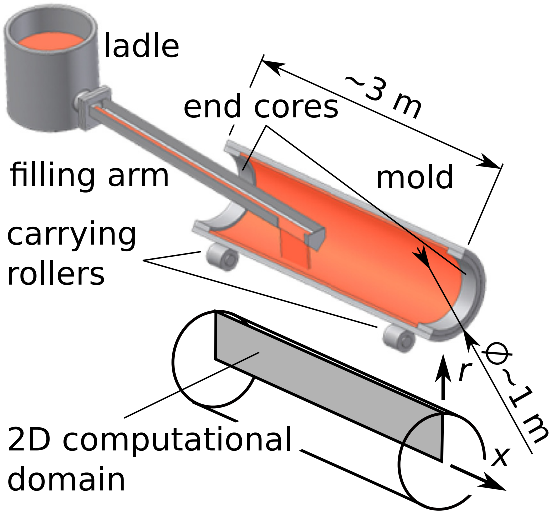

Fig. 1. A scheme of the horizontal centrifugal casting process with a computational domain and coordinates shown below it.

level-set [3,4], volume-of-fluid (VOF) [5], phase-field [6,7], particle method (marker-and-cell) [8], and the interface tracking [9]. Depending on the viewpoint of the observer—Eulerian or Lagrangian—interface capturing (VOF) and interface tracking approach can be distinguished respectively [10]. A main advantage of the earlier over the latter is that the topology of the interface is inherently treated, which allows for a description of much more complex interfaces. On the other hand, since the exact position of the interface is not exactly known, the treatment of boundary conditions, discontinuities across the interface and mass conservation still remain a challenge. For comparable grid sizes, interface tracking methods yield more accurate representation of the interface. Free-surface flows with less complicated interface topologies are generally solved by interface tracking methods, in which all grid points are treated either in a Lagrangian fashion or in an Arbitrary Lagrangian-Eulerian (ALE) approach [11], at which only the grid points close to the free-surface are being relocated. The ALE approach is advantageous when the liquid layer thickness goes to zero. In that case, the entire thickness of the layer is contained within a single cell along the vertical direction. Therefore, the 3D N–S equations collapse into the 2D shallow water equations (SWE) [12,13] due to the hydrostatic pressure assumption commonly applied within the interface cell.

In the present paper, a numerical model of horizontal centrifugal casting (HCC) process is introduced. In the HCC process [14–18], the liquid metal is poured inside a horizontal cylindrical mold rotating at high rates. Centrifugal forces push the liquid metal toward the wall of the mold with the radius R, resulting in a uniform thickness of the layer. The relatively cold mold extracts the heat from the liquid metal; therefore, solidification gradually proceeds toward the free-surface of the layer (Fig. 1). Often, when the liquid metal is partially solidified, an additional liquid of a different material is poured in. Most of the numerical studies solve the heat diffusion equation with a phase change source term. In order to account for the heat advection due to the flow, the thermal conductivity is artificially increased in the liquid region [19]. Several works in this area can be also found dedicated to the flow simulation, from which some of them rely on commercial CFD packages [20] and some on in-house codes, for example [21,22].

In the HCC, as a simple, nearly flat, free-surface and rather a weak effect of the surrounding air on dynamics of the liquid layer can be anticipated, an interface tracking approach is adopted here, inspired by Casulli [23–25] and further extended to account for two immiscible liquid layers. A robust finite difference-finite volume algorithm is derived from the non-hydrostatic N–S equations and it is suitable for structured and also unstructured grids provided the orthogonal layering of elements in the radial direction (Fig. 2). Due to the geometry configuration of the HCC, the cylindrical coordinates are used. Therefore, the axial, radial, and tangential axis notation can be seen throughout this paper. The pressure term is conveniently decomposed into the hydrostatic and the hydrodynamic part, which makes the algorithm very efficient especially when dealing with hydrostatic or nearly hydrostatic flows. The convective term and the axial viscous term are discretized explicitly using the reconstruction of the Lagrangian trajectory, especially popular in atmosphere modeling [26,27]. The resulting algorithm is mass conservative. In addition, when only a single layer of volume elements is considered, the algorithm degenerates into the shallow water equations. The proposed formulation can inherently handle drying and flooding of dry surfaces. In a subsequent step, the flow algorithm is followed by a stable finite volume scheme for the heat advection-diffusion equation with the solidification source term. Consequently, temperature differences result in thermal convection, which is in the N–S equations realized through a baroclinic pressure term.

In the next sections, the governing equations are firstly introduced, followed by detailed steps of the algorithm. Finally, results are presented in the form of numerical examples, some of them verified against temperature measurements from the plant and some against other numerical techniques.