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The optimal extended balanced loss function estimators

Selahattin Kaçıranlar [a,*], Issam Dawoud [b]

[a] Department of Statistics, Faculty of Sciences and Letters, Çukurova University, Adana, Turkey
[b] Department of Mathematics, Faculty of Science, Al-Aqsa University, Gaza, Palestine

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ABSTRACT

We derive the optimal heterogeneous, homogeneous and homogeneous unbiased estimators of the coefficient vector in a linear regression model under the extended balanced loss function of Shalabh et al. (2009). Risk functions and optimal predictors of the new estimators are evaluated and comparisons among the estimators are made with respect to the extended balanced loss function. Some of the theoretical results are illustrated by a numerical example. Moreover, the behavior of the proposed estimators is studied via a Monte-Carlo experiment in the sense of mean square error.

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1. Introduction

The ordinary least squares (OLS) estimator is the best linear unbiased estimator of regression parameters in a linear regression model. We can improve upon the variability of an estimator of a regression coefficient when the linearity and unbiasedness criteria are ignored. So, the minimum homogeneous and the minimum heterogeneous mean squared error estimators are introduced and discussed by Theil [1], Toutenburg [2], Rao [3], Theil [4, pp.125], Rao [5, pp. 305], Rao and Toutenburg [6, pp. 89–96] and Schaffrin [7–9] among others. Then, the minimum homogeneous and the minimum heterogeneous prediction mean squared error predictors related to the minimum homogeneous and the minimum heterogeneous mean squared error estimators are introduced and examined by Rao and Toutenburg [6, pp.159-164], Bibby and Toutenburg [10, pp. 76], Toutenburg [2,11–13], Goldberger [14] and Toutenburg and Trenkler [15] among others. Since the goodness of the fitted model criterion is quite often ignored where it is used to investigate the performance of estimators, Zellner [16] has introduced the balanced loss function (ZBLF) which focuses on estimates around true parameter and goodness of fit of model. Further, Shalabh [17] has presented a predictive loss function (PLF) that not only incorporates the ZBLF as its particular case but also measures the correlation between the goodness of fit of the model and concentration of estimates around the true parameter. The ZBLF has received considerable attention in the literature under different setups, for example, Rodrigues and Zellner [18], Giles et al. [19], Ohtani et al. [20], Ohtani [21,22] and Gruber [23]; see also Toutenburg and Shalabh [24] for the application of the ZBLF in some other areas of linear models. Appreciating the popularity of the ZBLF, Shalabh et al. [25] have extended it further and presented a general loss function, called an extended balanced loss function (EBLF), to evaluate the performance of the OLS and the Stein-rule estimation (SRE).

As a consequence, it appears reasonable to extend the idea of finding optimal estimators according to the general loss function, EBLF. So, we present some fundamental concepts and loss functions in Section 2. Then, in Section 3, we introduce and derive the optimal EBLF estimators and predictors and discuss their performances. In addition, we perform a simulation study and a numerical example in Section 3.

* Corresponding author.
E-mail address: skacir@cu.edu.tr (S. Kaçıranlar).

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2. Linear regression model with some concepts and the loss functions

Let us consider the following linear model:

$$y = X\beta + \varepsilon, \tag{1}$$

where y is an $n \times 1$ vector of responses, X is an $n \times p$ full column rank matrix of non-stochastic predetermined regressors, β is a $p \times 1$ vector of unknown parameters, and ε is an $n \times 1$ vector of $iid(0, \sigma^2)$ random errors.

The OLS estimator of β in model (1) is

$$\hat{\beta} = (X'X)^{-1}X'y. \tag{2}$$

This estimator is a widely used technique for estimating the linear regression models. The main reason for focusing on the OLS estimator is because it is unbiased and has the minimum variance among all linear unbiased estimators. However, if we ignore unbiasedness and consider biased estimators, we can propose improved estimators by employing matrix mean squared error (MSE) criterion.

In this context, the minimum MSE estimator is given by Theil [1]

$$\beta^* = \frac{\beta'X'y}{\sigma^2 + \beta'X'X\beta}\beta = \beta\beta'X'(X\beta\beta'X' + \sigma^2 \mathrm{I})^{-1}y. \tag{3}$$

The difficulty with β^* is that it is not an estimator indeed as it contains unknown parameters β and σ^2. In order to make it operational, Rao [5] suggested to insert prior values of β and σ^2 in β^*, while Farebrother [26] proposed to replace β and σ^2 by $\hat{\beta}$ and s^2 respectively, where

$$s^2 = \frac{1}{n-p}(y - X\hat{\beta})'(y - X\hat{\beta}). \tag{4}$$

So, the operational version of the optimal estimator β^* is obtained as

$$\hat{\beta}^* = \frac{\hat{\beta}'X'y}{s^2 + \hat{\beta}'X'X\hat{\beta}}\hat{\beta} = \hat{\rho}\hat{\beta} \tag{5}$$

where $\hat{\rho} = \frac{\hat{\beta}'X'y}{s^2 + \hat{\beta}'X'X\hat{\beta}}$. Dwivedi and Srivastava [27] worked out the large sample properties of $\hat{\beta}^*$.

As another improved estimator, Stahlecker and Trenkler [28] proposed to utilize prior information for estimating β with respect to MSE criterion in which the estimator is given as

$$\beta_{opt} = \rho\beta + (1 - \rho)c \tag{6}$$

where $\rho = \frac{\beta'X'y}{\sigma^2 + \beta'X'X\beta}$ and c is a $n \times 1$ vector of values which reflects our a prior belief on β, which could be some past values of β or even a guess about β. The estimator β_{opt} is also a minimum MSE estimator and may be called as the best heterogeneous linear estimator of β. However, β_{opt} is not of any practical utility despite its optimality. Therefore, Stahlecker and Trenkler [28] proposed to replace β and σ^2 by $\hat{\beta}$ and s^2 respectively as done by Farebrother [26] for β^*. This gives the operational variant of β_{opt} as

$$\hat{\beta}_{ST} = \hat{\rho}\hat{\beta} + (1 - \hat{\rho})c. \tag{7}$$

The estimator $\hat{\beta}_{ST}$ relies on the prior information. This prior information may be of two types:

(i) Some past values on β, and/or
(ii) A guess about β based on past experience.

In both cases, the prior vector c may or may not conform to the current sample information. So, a possible remedy is to take c as a stochastic vector instead of a fixed one. A simple choice is

$$\overline{\beta} = \frac{l'\hat{\beta}}{l'l} = \frac{1}{p}\sum_{i=1}^{p}\hat{\beta}_i, \text{ where } l = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}_{p\times 1} = 1_p. \tag{8}$$

Shrinking $\hat{\beta}_{ST}$ towards the mean vector $\overline{\beta}l$ would be more useful than shrinking it towards the fixed vector c when $\hat{\rho} \to 0$. This leads to the estimator

$$\hat{\beta}_{TS} = \hat{\rho}\hat{\beta} + (1 - \hat{\rho})\overline{\beta}l. \tag{9}$$

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