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Junjie Ma, Huilan Liu

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A well-conditioned Levin method for calculation of highly oscillatory integrals and its application☆

Junjie Ma[a,b,*], Huilan Liu[a,b,*]

[a]School of Mathematics and Statistics, Guizhou University, Guiyang, Guizhou 550025, P.R. China.
[b] Guizhou Provincial Key Laboratory of Public Big Data, Guizhou University, Guiyang, Guizhou 550025, P.R. China.

Abstract

This paper is devoted to studying efficient calculation of generalized Fourier transform $\int_{-1}^{x} f(t)e^{i\omega g(t)}dt$. For the general phase function $g(t)$, we develop a modified Levin method by the spectral coefficient approach. A sparse and well-conditioned linear system is constructed to help accelerate calculation of highly oscillatory integrals. Numerical examples are included to show the convergence properties of the new method with respect to both quantities of collocation points and the frequency ω. Futhermore, we apply this approach to solving oscillatory Volterra integral equations.

Keywords: Spectral coefficient method, highly oscillatory integral, Levin method, numerical integration, Chebyshev polynomial.

1. Introduction

Calculation of highly oscillatory integrals (HOIs) is an everlasting topic in computational mathematics, for their wide applications in acoustic scattering ([4]), circuit simulation ([7]), calculation of mutual impedance between conductors ([32]), and so on. Consider calculation of a class of generalized Fourier transforms, such as

$$I_1 = \int_{-1}^{x} f(t)e^{i\omega g(t)}dt, \; x \in [-1,1]. \tag{1}$$

Here the amplitude function $f(t)$ and the phase function $g(t)$ are given on $[-1,1]$, and $\omega \gg 1$. Generally, such integrals cannot be computed in closed-forms, which implies one has to resort to numerical integration. A direct approach to HOI (1) is to discretize it by a

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*Corresponding author.
E-mail address: jjma@gzu.edu.cn, majunjie.math@gmail.com