A modified numerical scheme and convergence analysis for fractional model of Lienard's equation

Devendra Kumar, Ravi P. Agarwal, Jagdev Singh

PII:	S0377-0427(17)30130-9
DOI:	http://dx.doi.org/10.1016/j.cam.2017.03.011
Reference:	CAM 11058

To appear in: *Journal of Computational and Applied Mathematics*

Received date: 2 February 2017
Revised date: 3 March 2017

A MODIFIED NUMERICAL SCHEME AND CONVERGENCE ANALYSIS FOR FRACTIONAL MODEL OF LIENARD'S EQUATION

DEVENDRA KUMAR*, RAVI P. AGARWAL AND JAGDEV SINGH

ABSTRACT. The key purpose of the present work is to constitute a numerical algorithm based on fractional homotopy analysis transform method to study the fractional model of Lienard's equations. The Lienard's equation describes the oscillating circuits. The suggested scheme is a merger of homotopy analysis technique, classical Laplace transform and homotopy polynomials. The uniqueness and convergence analysis of the solution is also discussed. The numerical and graphical results elucidate that the suggested approach is very straightforward and accurate.

1. INTRODUCTION

The Lienard's equation is a nonlinear second order differential equation proposed by Lienard [1] and is presented as

$$x'' + a_1(x)x' + a_2(x) = a_3(t), \tag{1.1}$$

where $a_1(x)x'$ indicates the damping force, $a_2(x)$ denotes the restoring force and $a_3(t)$ stands for the external force. In the development of radio and vacuum tube technology, Lienard equations were extremely investigated as they can be utilized to describe the oscillating circuits. The Lienard's equation contains the damped pendulum or a damped spring-mass system as a particular case. It is also very useful as nonlinear models of various scientific fields while considering various selection of $a_1(x)$,$a_2(x)$ and $a_3(t)$. For instance, the choice $a_1(x) = \varepsilon(x^2 - 1)$,$a_2(x) = x$ and $a_3(t) = 0$ leads Eq. (1.1) to the Van der Pol equation used as a nonlinear model of electronic oscillation. The particular case of Eq. (1.1) is written as

$$x'' + \lambda x' + \mu x^3 + \nu x^5 = 0. \tag{1.2}$$

where λ, μ and ν are real coefficients. The Eq. (1.2) was studied by many researchers, notably Feng [2], Kong [3], Matinfar et al. [4,5], and others.
We recall that the nature of the trajectory of the fractional derivatives is non-local interpreted fractional derivative as the index of memory [6-10]. As a result any physical process explained with the help of fractional operators has a memory effect, i.e. future state of a system depends not only on present state but also depends upon past states [11-15]. Consequently, the Lienard's equation (1.2) involving the derivatives of arbitrary order would be a generalization of the Lienard's equation (1.2). Thus the fractional

2010 *Mathematics Subject Classification.* 34A08, 35A20, 35A22.
* Corresponding author.