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Efficient heuristics for the minimum labeling global cut problem

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Abstract

Let $G = (V, E, L)$ be an edge-labeled graph. Let V be the set of vertices of G, E the set of edges, L the set of labels (colors) such that each edge $e \in E$ has an associated label $L(e)$. The goal of the minimum labeling global cut problem (MLGCP) is to find a subset $L' \subseteq L$ of labels such that $G' = (V, E', L \backslash L')$ is not connected and $|L'|$ is minimized. In this work, we generate random instances for the MLGCP to perform empirical tests. Also propose a set of heuristics using concepts of Genetic Algorithm and metaheuristic VNS, including some of their procedures, like two local search moves, and an auxiliary data structure to speed up the local search. Computational experiments show that the metaheuristic VNS outperforms other methods with respect to solution quality.

Keywords: Edge-Labeled Graphs, Variable Neighborhood Search, Connectivity

1 Introduction

In the last few years, several papers have addressed problems defined over an edge-labeled graph (ELG). As opposed to weighted graphs, every edge of an

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ELG has an associated label instead of a weight. ELGs are used to model qualitative characteristics of different regions of some graph. It is common to use colors to represent labels in ELGs, and for this reason, the terms label and color are used interchangeably in this work.

This work addresses the minimum labeling global cut problem (MLGCP). The MLSGCP is defined as follows: given an ELG, find the minimum number of labels such that their removal turns the graph into a disconnected one. Figure 1 illustrates the problem: Figure 1(a) describes the input graph; Figure 1(b) shows a label cut with the colors E and F; and Figure 1(c) emphasizes that the graph is disconnected if these labels are removed.

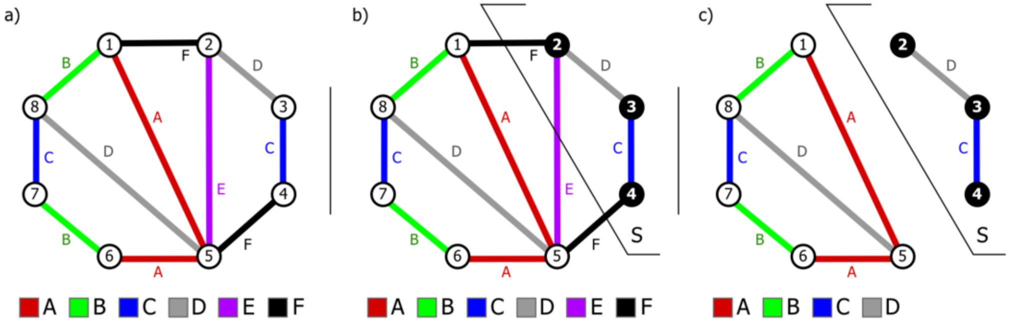

Fig. 1. Example of the minimum labeling global cut problem

Formally, let $G = (V, E, L)$ be a simple and undirected ELG such that V represents the set of vertices of G, E the set of edges and L the set of labels. Additionally every edge $e \in E$ has the associated label $L(e)$. The goal of the MLGCP is to find a set of labels $L' \subseteq L$ with minimum cardinality such that the graph $G = (V, E', L \backslash L')$ is disconnected, where $E' = \{e \mid L(e) \in L \backslash L'\}$.

Let S be a proper subset of V, $\overline{S}$ be the complement of S, and let $C = (S, \overline{S})$, be a partition of V. C is an s-t cut if $s \in S$ and $t \in \overline{S}$. The problem of finding an s-t cut-set with the minimum number of labels is NP-complete [1]. Besides that, the complexity of the MLGCP remains a theoretically open question [5]. Intuitively, we believe that the MLGCP is probably NP-hard.

Main motivation for studying the MLGCP is to measure connectivity of a network when its links have shared risks of failure, as addressed by [2] and [1]. For instance, all wireless links within a certain frequency can be shut down by generating strong noise. Another example is when two or more link cables use the same physical path in some part of their way. [6] demonstrates that the MLGCP is polynomial time solvable if the input graph is planar, have bounded treewidth or if every label is represented in a small number of edges.