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On integrating an Iterated Variable Neighborhood Search within a bi-objective Genetic Algorithm: Sum Coloring of Graphs Case Application

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Abstract

The minimum sum coloring of graphs is a variant of the classical graph coloring problem which is known to be NP-hard. The problem consists on minimizing the sum colorings of different graph vertices. In this paper, we propose a new bi-objective model for the underlying problem. We also propose for the resolution a hybrid schema which combines a bi-objective genetic algorithm with an Iterated Variable Neighborhood Search. The proposed approach relies on the use of different dedicated evolutionary operators mainly crossover and mutation. We also note two important features of the Variable Neighborhood Search: the use of destroy/repair method for shaking step and a multi-neighborhood search. Combined methods led us to preliminary promising results.

Keywords: Minimum Sum Coloring Problem, Hybrid search, Bi-objective modelization, Variable Neighborhood Search, VEGA algorithm, Local optimization.

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1 Introduction

The Minimum Sum Coloring Problem (MSCP) was firstly introduced by Kubicka in 1989 [9]. It has received a lot of attention in the past few years since it can model several interesting practical problems in scheduling and ressource allocation [[1], [5]]. The problem could be modeled based on a simple undirected graph G=(V, E) where V is the set of $|V| = n$ vertices and E the set of $|E| = m$ edges. Let $\{1, \ldots, k\}$ be a set a colors. A valid **k-coloring** of the graph G is an application $c : V \rightarrow \{1, \ldots, k\}$ such as $c(x) \neq c(y)$, $\forall (x, y) \in E$. However, if $c(x) = c(y)$, vertices x and y are then in **conflict** or **conflicting nodes**.

From another point of view, a proper k-coloring could also be considered as a partition of V into k disjoint independent sets or **colors classes**$\{V_1, \ldots, V_k\}$. In graph theory, an independent set is defined as a set of vertices in a graph with no adjacent nodes. Color class V_i is usually defined by its cardinality which corresponds to the number of its vertices denoted by $|V_i|$. The smallest integer k for which a proper coloring of a graph exists is called the **chromatic number** and it is denoted by $\chi(G)$. Looking for a proper coloring using χ colors is the **graph coloring problem**. However, finding the k-coloring is known as **k-coloring problem**. The minimum sum coloring problem is to find a valid coloring of G using naturel numbers such that the folllowing sum of colors is minimized:

$$f(c) = \sum_{i=1}^{n} c(v_i) \quad (1)$$

where $c(v_i)$ is the color of the vertice v_i. This minimal sum is called **chromatic sum** and denoted by $\sum(G)$. The minimal number of colors used in obtaining the chromatic sum is called **strength** $s(G)$ of the graph. Note that $\chi(G)$ is a lower bound of $s(G)$, i.e. $\chi(G) \leq s(G)$.

As the minimum sum coloring problem is proven to be NP-hard [10], there exist approximation algorithms only for specific graphs [[10], [2], [8], [3]]. Therefore, much effort has been devoted for proposing heuristic and metaheuristic algorithms in order to compute upper or lower bounds of the chromatic sum. These methods belong to three classes: greedy algorithms, local search heuristics and evolutionary algorithms. For more details about the aforementioned algorithms, one can refer to [7]. From the literature, we note that operating with non feasible solutions leads to promising results even with a very simple schema of resolution. This strategy provides more "freedom" for the search in order to better explore the search space of the problem. In this work, we adopt the same strategy during our search process. Indeed, we propose a