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Line Signed Graph of a Signed Total Graph

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Abstract

A *signed total graph* is an ordered pair $T_\Sigma(\Gamma(R)) := (T(\Gamma(R)),\ \sigma)$, where $T(\Gamma(R)$ is the *total graph* of a commutative ring R, called the underlying graph of $T_\Sigma(\Gamma(R))$ and $T_\Sigma(\Gamma(R))$ is associated with a signing of its edges (a, b) by the function σ such that $\sigma(a, b)$ is '+' if $a \in Z(R)$ or $b \in Z(R)$ and '$-$' otherwise. The aim of this paper is to gain a deeper insight into the notion of signed total graph by characterizing the rings for which line signed graph $L(T_\Sigma(\Gamma(R)))$ of signed total graph are $\mathcal{C}$-consistent, $T_\Sigma(\Gamma(R))$-consistent and sign-compatible.

Keywords: finite commutative rings, canonical marking, signed graph, total graph.

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1 Introduction

We assume throughout that all rings are finite commutative with $1 \neq 0$. Let R be a commutative ring with $Reg(R)$ its set of regular elements and $Z(R)$ be its set of zero-divisors. As in [3], we define the *total graph* of a commutative ring R, denoted by $T(\Gamma(R))$ is a graph whose vertex set $V = R$ and two distinct vertices x and y are said to be adjacent if and only if $x+y \in Z(R)$. The study of $T(\Gamma(R))$ breaks into two cases depending on whether or not $Z(R)$ is an ideal of R. Thus, the regular graph of R, $Reg(\Gamma(R))$, is an induced subgraph of $T(\Gamma(R))$ on the regular elements of R, and the girth of total graph and regular graph of a commutative ring are contained in the set $\{3, 4, \infty\}$ (for more details see [3]). For the case when $Z(R)$ is an ideal of R, the following interesting result established in [3] provides some insight into the structure of total graphs, and will be frequently used in this paper:

Lemma 1.1 ([3, Theorem 2.2]). *Let R be a commutative ring such that $Z(R)$ is an ideal of R, and let $|Z(R)| = \alpha$ and $|R/Z(R)| = \beta$, where α, β are positive integers. Then*

$$T(\Gamma(R)) \cong \begin{cases} K_\alpha \cup \underbrace{K_\alpha \cup K_\alpha \cup \cdots \cup K_\alpha}_{(\beta-1)-copies} & \text{if } 2 \in Z(R), \\ K_\alpha \cup \underbrace{K_{\alpha,\alpha} \cup K_{\alpha,\alpha} \cup \cdots \cup K_{\alpha,\alpha}}_{(\frac{\beta-1}{2})-copies} & \text{if } 2 \notin Z(R). \end{cases} \quad (1)$$

Pranjali and Acharya [11] initiated the study of *signed total graph* of commutative ring with the motivation to study the total graph in the realm of signed graph. Moreover, they have characterized the commutative rings for which the signed total graph $T_\Sigma(\Gamma(R))$ and its line signed graph $L(T_\Sigma(\Gamma(R)))$ are balanced. The formal definition of the new notion introduced in [11] is as follows:

Definition 1.2 A signed total graph is an ordered pair $T_\Sigma(\Gamma(R)) := (T(\Gamma(R)), \sigma)$, where $T(\Gamma(R)$ is the total graph of a commutative ring R and for each

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