Topology and its Applications 241 (2018) 70–77

Contents lists available at ScienceDirect

Topology and its Applications

www.elsevier.com/locate/topol

Virtual Special Issue – TOPOSYM 2016

On the estimation of the large inductive dimension of a product of compacta ☆

D.N. Georgiou [a,*], K.L. Kozlov [b]

[a] *University of Patras, Department of Mathematics, 265 04 Patras, Greece*
[b] *Lomonosov Moscow State University, Faculty of Mechanics and Mathematics, Russia*

A R T I C L E   I N F O

Article history:
Received 19 December 2016
Received in revised form 7 August 2017
Accepted 4 December 2017
Available online 27 March 2018

MSC:
54B99
54C25

Keywords:
Large inductive dimension
Product space
Normal base
Base dimensions I

A B S T R A C T

We prove the analog of Pasynkov's result on finite-dimensionality of topological products for the dimension-like function I defined by S. Iliadis. Let $\mathcal{F}_j$ be a normal base on a topological space X_j, $j = 1, 2$. Then $\mathrm{I}(X_1 \times X_2, \mathcal{F}_1 \otimes \mathcal{F}_2) \leq \varphi(\mathrm{I}(X_1, \mathcal{F}_1), \mathrm{I}(X_2, \mathcal{F}_2))$, where φ is a recursion relation. As a consequence we get the result on the finite-dimensionality of topological products of compacta for the large inductive dimension.

1. Introduction

One of the central questions in dimension theory is the question of estimation of dimension of products. See [2], [6] and [15] for details. In the case of the large inductive dimension of compacta there are the result of V. V. Filippov [7] which demonstrates the non-fulfillment of the inequality

$$\mathrm{Ind}(X \times Y) \leq \mathrm{Ind}X + \mathrm{Ind}Y,$$

and the result of B. A. Pasynkov [16] which states, among other things, that the product of finite dimensional compacta is finite dimensional (see, also [14] where this result is given without proof). Moreover, the recursion

☆ Work is supported by the Erasmus Programme of the University of Patras.
* Corresponding author.
E-mail addresses: georgiou@math.upatras.gr (D.N. Georgiou), kkozlov@mech.math.msu.su (K.L. Kozlov).

https://doi.org/10.1016/j.topol.2018.03.030
0166-8641/

relation for the function that estimates the dimension of the normal product of a nonempty normal and compact spaces by the dimensions of its factors is given there:

$$\mathrm{Ind}(X \times Y) \le d(\mathrm{Ind}X,\ \mathrm{Ind}Y),$$

where $d : \omega \times \omega \to \omega$ is a map such that $d(0, m) = m$ if $m \in \omega$, $d(m, 0) = (m + 1)d(m - 1, 0) + 1$ if $m > 0$, $d(k, m) = (k + 1)(d(k - 1, m) + d(k, m - 1)) + 1$ if $k, m > 0$.

In the case of compacta the estimation function can easily be modified. Let $d' : \omega \times \omega \to \omega$ be a map such that $d'(0, m) = d'(m, 0) = m$ if $m \in \omega$, $d'(k, m) = (\min\{k, m\} + 1)(d'(k - 1, m) + d'(k, m - 1)) + 1$ if $k, m > 0$. So, we have $d'(n, m)$:

m / n	0	1	2	3	...
0	0	1	2	3	...
1	1	5	15	37	...
2	2	15	91		...
3	3	37			...
...	...	...	...	...	...

Recently in [13] the exact value of the dimension of the product of compacta in Filippov's example was given. This result was obtained with the usage of the dimension-like function I of a space by a normal base which was introduced by S. Iliadis [11]. In the present paper we prove the analog of Pasynkov's result on finite-dimensionality of topological products for the dimension-like function I. As a consequence we get the result on the finite-dimensionality of topological products of compacta for the large inductive dimension.

$$\mathrm{Ind}(X \times Y) \le \varphi(\mathrm{Ind}X,\ \mathrm{Ind}Y),$$

where $\varphi : \omega \times \omega \to \omega$ is a map such that $\varphi(0, m) = \varphi(m, 0) = m$ if $m \in \omega$, $\varphi(1, 1) = 2$, $\varphi(1, m) = \varphi(m, 1) = 2\varphi(1, m - 1) + m + 1$ if $m > 1$, $\varphi(n, m) = (\min\{n, m\} + 1)(\varphi(n - 1, m) + \varphi(n, m - 1)) + 1$ if $n > 1$, $m > 1$.

So, we have $\varphi(n, m)$:

m / n	0	1	2	3	4	...
0	0	1	2	3	4	...
1	1	2	7	18	41	...
2	2	7	43	184		...
3	3	18	184	1473		...
4	4	41				...
...	...	...	...	...	...	...

All spaces are assumed to be Tychonoff, maps are continuous and notations, terminology and designations are from [5]. By a neighborhood we always understand an open neighborhood and cl$A$ is the closure of a subset A of a space X.

2. Preliminaries

The notion of a normal base for the closed subsets of a space was defined in [8]. The existence of such bases characterizes Tychonoff spaces. Natural normal bases are the following:

(a) the family of all closed subsets of a normal space;
(b) the family $Z(X)$ of all zero-sets of a completely regular space X; and