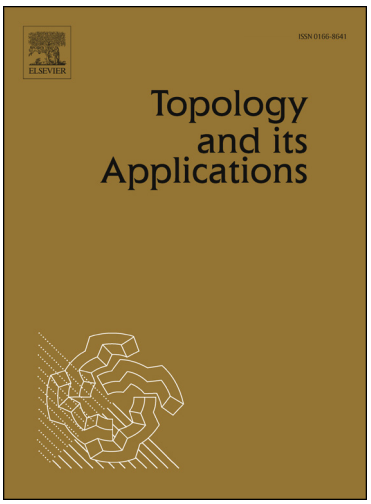
Accepted Manuscript



Sober metric approach spaces

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PII: S0166-8641(17)30556-4
DOI: https://doi.org/10.1016/j.topol.2017.10.019
Reference: TOPOL 6297

To appear in: *Topology and its Applications*

Received date: 6 July 2016
Revised date: 18 June 2017
Accepted date: 24 October 2017

Please cite this article in press as: W. Li, D. Zhang, Sober metric approach spaces, *Topol. Appl.* (2017), https://doi.org/10.1016/j.topol.2017.10.019

Sober metric approach spaces[☆]

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Abstract

It is proved that a metric space is sober, as an approach space, if and only if it is Smyth complete.

Keywords: Metric space, Yoneda completeness, Smyth completeness, approach space, metric approach space, sober approach space
2010 MSC: 18B30, 18B35, 54B30, 54E99

1. Introduction

Approach spaces, introduced by Lowen [21], are a common extension of topological spaces and metric spaces. By a metric on a set X we understand, as in Lawvere [19], a map $d : X \times X \to [0, \infty]$ such that $d(x, x) = 0$ and $d(x, y) + d(y, z) \geq d(x, z)$ for all $x, y, z \in X$. An extensive investigation of approach spaces can be found in the monographs of Lowen [22, 23]. An approach space is said to be a topological one if it is generated by a topological space; and it is said to be a metric one if it is generated by a metric space.

Sober approach spaces, a counterpart of sober topological spaces in the metric setting, are introduced in [1]. It is proved there that a topological space is sober as an approach space, if and only if it is sober as a topological space. So, it is natural to ask what kind of metric approach spaces are sober? A partial answer is obtained in [1]. If d is a usual metric (i.e., a symmetric, separated and finitary metric) on a set X, it follows from Corollary 5.19 in [1] that (X, d) is sober, as an approach space, if and only if (X, d) is a complete metric space. This paper presents a complete answer to this question. The answer is a bit surprising: a metric space is sober, as an approach space, if and only if it is Smyth complete. A metric space is Smyth complete if every forward Cauchy net in it converges in its symmetrization [7, 16]. Smyth completeness originated in the works of Smyth [26, 27] that aimed to provide a common framework for the domain approach and the metric space approach to semantics in computer science.

As advocated in [8, 9, 10], in this paper we emphasize that the relationship between approach spaces and metric spaces is analogous to that between topological spaces and ordered sets. This point of view has proved to be fruitful, and is well in accordance with the thesis of Smyth [26] "that domains are, or should be, a prime area for the application of quasi-uniform ideas, and can help us to get the definitions right."

[☆]This work is supported by National Natural Science Foundation of China (11371265).
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