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Axiomatizations of team logics

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ABSTRACT

In a modular approach, we lift Hilbert-style proof systems for propositional, modal and first-order logic to generalized systems for their respective team-based extensions. We obtain sound and complete axiomatizations for the dependence-free fragment FO(∼) of Väänänen's first-order team logic TL, for propositional team logic PTL, quantified propositional team logic QPTL, modal team logic MTL, and for the corresponding logics of dependence, independence, inclusion and exclusion. As a crucial step in the completeness proof, we show that the above logics admit, in a particular sense, a semantics-preserving elimination of modalities and quantifiers from formulas.

1. Introduction

While their history goes back to ancient philosophers, propositional and modal logics have assumed an outstanding role in the age of modern computer science, with plentiful applications in software verification, modeling, artificial intelligence, and protocol design. An important property of a logical framework is *completeness*, i.e., that the act of mechanical reasoning can effectively be done by an algorithm. The question of completeness of first-order logic, which is the foundation of today's mathematics, was not settled until the famous result of Gödel in 1929. Until today, the area of proof theory has achieved tremendous progress and is still a growing field, especially with regard to many variants of propositional and modal logics as well as non-classical logics (see e.g. Fitting [3]).

A recent extension of classical logics is so-called *team logic*. It originated from the concept of quantifier dependence and independence. The following question has been long-known in linguistics: how can the statement

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For every x there is y, and for every u there is v such that $P(x,y,u,v)$.

be formalized in first-order logic such that y and v are chosen independently? Some suggestions were Henkin's *branching quantifiers* [15] as well as *independence-friendly logic* IF by Hintikka and Sandu [16]. The idea of the latter is to assert dependence and independence between quantifiers syntactically, implemented semantically by a game of imperfect information. Hodges [17] proved that IF also admits a compositional semantics if formulas were evaluated on *teams*, which are sets of assignments, instead of single assignments. In this vein, Väänänen [28] introduced *dependence logic* D. Here, the fundamental idea is that dependencies are not stated alongside the quantifiers, but instead are expressed as logical *dependence atoms*, written $=(x, y)$, which means "x functionally determines y."

Beside Väänänen's dependence atom, a variety of atomic formulas solely for reasoning in teams were introduced. Galliani [5] as well as Grädel and Väänänen [8] pointed out connections to database theory; they formalized common constraints like *independence* $\perp$, *inclusion* $\subseteq$ and *exclusion* $\mid$ as atoms in the framework of team semantics. Beside first-order logic, all these atoms have also been adapted for modal logic ML [27], and (quantified) propositional logic PL resp. QPL [11,26,32].

As for any logic, the question of axiomatizability arises for these logics with team semantics, in particular for the extensions of first-order logic. However, dependence logic D is as expressive as existential second-order logic SO($\exists$) [28], while its extension TL, obtained from D by adding a semantical negation $\sim$, is equivalent to full second-order logic SO [20]. Accordingly, both are non-axiomatizable. Later, Kontinen and Väänänen [21] gave a partial axiomatization in the sense that FO-consequences of D-formulas are derivable, and recently a system that can derive all so-called *negatable* D-formulas was presented by Yang [30].

For certain fragments of propositional and modal team logic, axiomatizations exist. Hannula [10] presented natural deduction systems for propositional dependence logic PDL, quantified propositional dependence logic QPDL and extended modal dependence logic EMDL. By contrast, Sano and Virtema [26] gave Hilbert-style axiomatizations and labeled tableau calculi for propositional dependence logic PDL and (extended) modal dependence logic (E)MDL. Independently, Yang [31] presented both Hilbert-style axiomatizations and natural deduction systems for a family of so-called *downward-closed* modal logics with team semantics, which includes EMDL as well.

However, a fundamental restriction of these solutions is that they all rely on the absence of Boolean negation. As a consequence, team logics with negation, most notably propositional team logic (PTL), modal team logic (MTL) and FO($\sim$), require a different approach.

Contribution

In this paper, we present complete axiomatizations for several team logics including the $=(\cdot,\cdot)$-free fragment of TL, coined FO($\sim$) by Gallani [6]. Here, we consider it under *lax semantics* [5].

By showing that FO($\sim$) is axiomatizable, we identify the dependence atom $=(\cdot,\cdot)$, and not team semantics itself, as the source of incompleteness of D and TL. One interpretation is that reasoning about teams can be axiomatized; but only if we cannot talk about the internal dependencies between the elements of the team.

A crucial step in the completeness proof is the perhaps surprising fact that TL without $=(\cdot,\cdot)$ collapses to $\mathcal{B}$(FO), the Boolean closure of classical first-order logic FO under team semantics. The latter has the so-called *flatness property*, which implies that any classical proof system of FO is also adequate for team semantics. From there, an axiomatization of $\mathcal{B}$(FO) is easily found in a similar way as for propositional logic.

Whether logics not collapsing to $\mathcal{B}$(FO) have axiomatizations is beyond the scope of this paper. Our approach, however, also yields results for (quantified) propositional and modal team logics. They can define their atoms of dependence, independence and inclusion in terms of other connectives, whereas this is not possible in the first-order setting. For this reason, the logics QPTL, PTL and MTL collapse to $\mathcal{B}$(QPL), $\mathcal{B}$(PL) and $\mathcal{B}$(ML) in a similar fashion as FO($\sim$) to $\mathcal{B}$(FO), and we obtain complete axiomatizations as a byproduct. Fig. 1 illustrates this.