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Intuitionistic nonstandard bounded modified realisability and functional interpretation ☆

Bruno Dinis [a,1], Jaime Gaspar [b,c,*,2]

[a] *Departamento de Matemática, Faculdade de Ciências da Universidade de Lisboa, Campo Grande, Edifício C6, 1749-016 Lisboa, Portugal*
[b] *School of Computing, University of Kent, Canterbury, Kent, CT2 7NF, United Kingdom*
[c] *Centro de Matemática e Aplicações (CMA), FCT, UNL, Portugal*

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ABSTRACT

We present a bounded modified realisability and a bounded functional interpretation of intuitionistic nonstandard arithmetic with nonstandard principles.
The functional interpretation is the intuitionistic counterpart of Ferreira and Gaspar's functional interpretation and has similarities with Van den Berg, Briseid and Safarik's functional interpretation but replacing finiteness by majorisability.
We give a threefold contribution: constructive content and proof-theoretical properties of nonstandard arithmetic; filling a gap in the literature; being in line with nonstandard methods to analyse compactness arguments.

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* Corresponding author.
E-mail addresses: bmdinis@fc.ul.pt (B. Dinis), mail@jaimegaspar.com, jg478@kent.ac.uk (J. Gaspar).
URLs: http://www.jaimegaspar.com, http://www.cs.kent.ac.uk/people/rpg/jg478 (J. Gaspar).

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1. Introduction

In the past few years there has been a growing interest in the study of nonstandard arithmetic using realisabilities and functional interpretations. Particularly interesting for us are:

1. Ferreira and Gaspar's functional interpretation [6] (which deals with majorisability);
2. Van den Berg, Briseid and Safarik's realisability and functional interpretation [2] (which deals with finiteness).

In their spirit we present:

1. a realisability (based on Ferreira and Nunes's bounded modified realisability [7], and similar to Van den Berg, Briseid and Safarik's realisability [2]);
2. a functional interpretation (based on Ferreira and Oliva's functional interpretation [8], and Ferreira and Gaspar's functional interpretation [6], and similar to Van den Berg, Briseid and Safarik's functional interpretation [2]);

of intuitionistic nonstandard arithmetic with nonstandard principles and we prove their soundness and characterisation theorems.

Overall we give a threefold contribution:

1. we give as applications various results, for example about
   (a) the constructive content (such as bounded-program extraction and extraction of bounds on witnesses);
   (b) proof-theoretical properties (such as relative consistency results and independence results);
   of nonstandard arithmetic;
2. we fill an existing gap in the literature, for example
   (a) we show that if we restrict our realisability to the "purely external fragment" (where there are only quantifiers of the form $\exists^{\mathrm{st}}$ and $\forall^{\mathrm{st}}$), then we recover Ferreira and Nunes's bounded modified realisability, and analogously if we restrict our functional interpretation to the "purely external fragment", then we recover Ferreira and Oliva's bounded functional interpretation;
   (b) we think that if we combine our intuitionistic functional interpretation with a suitable negative translation, then we obtain Ferreira and Gaspar's classical functional interpretation (a result that we hope to publish soon);
3. we are in line with the argument that nonstandard methods can be used to analyse compactness arguments [8,7,6,5].

Our realisability and functional interpretation, like previous ones, make use of Nelson's syntactic approach to nonstandard analysis called internal set theory [12,13] by extending the language with a new unary predicate $\mathrm{st}(t)$ (meaning "t is standard").

2. Framework

Let $\mathsf{E\text{-}PA}^{\omega}$ and $\mathsf{E\text{-}HA}^{\omega}$ be (respectively) Peano and Heyting arithmetics in all finite types with full extensionality and with primitive equality only at type 0. In the next definition, proposition and theorem, we recall well-known facts about the Howard–Bezem strong majorisability $\leq^{*}_{\sigma}$ [3,9].

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