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The quenching behavior of a quasilinear parabolic equation with double singular sources

Le comportement désactivant d'une équation parabolique quasi linéaire avec deux sources singulières

Liping Zhu

College of Science, Xi'an University of Architecture & Technology, Xi'an, 710055, China

A R T I C L E   I N F O

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A B S T R A C T

In this paper, we study the quenching behavior for a one-dimensional quasilinear parabolic equation with singular reaction term and singular boundary flux. Under certain conditions on the initial data, we show that quenching occurs only on the boundary in finite time. Moreover, we derive some lower and upper bounds of the quenching rate and get some estimates for the quenching time.

R É S U M É

Nous étudions ici le comportement désactivant d'une équation parabolique quasi-linéaire avec un terme de réaction singulier et un flux au bord singulier. Sous certaines conditions sur les données initiales, nous montrons que la désactivation intervient seulement au bord en temps fini. De plus, nous obtenons des bornes inférieure et supérieure du taux de désactivation ainsi que des estimations du temps de désactivation.

1. Introduction

In this paper, we mainly study the following problem with double singular sources

$$\begin{cases} u_t = \left(|u_x|^{p-2}u_x\right)_x - u^{-r}, & 0<x<1,\ t>0, \\ u_x(0,t) = u^{-q}(0,t),\ \ u_x(1,t)=0, & t>0, \\ u(x,0)=u_0(x), & 0\le x\le 1, \end{cases} \tag{1.1}$$

E-mail address: 78184385@qq.com.

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where $p > 1$, $r, q > 0$, and u_0 satisfies the second-order compatibility conditions. Equation (1.1) models a generalized electrostatic Micro-Electro-Mechanical-System (MEMS) device consisting of a thin dielectric elastic membrane. In this model where $p = 2$, the dynamic solution u characterizes the dynamic deflection of the elastic membrane; we refer the reader to [3,9] and the references therein. If quenching occurs in finite time, we denote by T the quenching time, or else $T = \infty$. Many authors have studied quenching problems with various nonlinear source terms and boundary conditions, we refer to [1,4–6,8,10–12,17] and references therein. Zhao [17] considered the problem

$$\begin{cases} u_t = \Delta u + u^p, & x \in \Omega,\ t > 0, \\ \dfrac{\partial u}{\partial \nu} = -u^{-q}, & x \in \partial\Omega,\ t > 0, \\ u(x,0) = u_0(x), & x \in \Omega. \end{cases} \tag{1.2}$$

Under certain conditions on initial data, Zhao not only showed that quenching occurs only on the boundary, but also derived the quenching rate;

$$\min_{x \in \partial\Omega} u(x,t) \sim (T-t)^{\frac{1}{2(q+1)}}, \quad t \to T^-.$$

In [18,19], Zhi and Mu studied the following semilinear equation

$$\begin{cases} u_t = u_{xx} + f(x)(1-u)^{-p}, & 0 < x < 1,\ t > 0, \\ u_x(0,t) = u^{-q}(0,t),\ u_x(1,t) = 0, & t > 0, \\ u(x,0) = u_0(x), & 0 \le x \le 1. \end{cases} \tag{1.3}$$

They proved that quenching occurs only at $x = 0$, and that the quenching rate satisfies $u(0,t) \sim (T-t)^{1/[2(q+1)]}$, as $t \to T^-$. While $f(x) \equiv 1$ and the boundary flux becomes $u_x(1,t) = -u^{-q}(1,t)$, Selcuk and Ozalp [11] showed that the lower bound of the quenching rate is $u(0,t) \ge 1 - C(T-t)^{1/(p+1)}$ for t sufficiently close to T. Furthermore, Ozalp and Selcuk [8] studied the semilinear equation with singular reaction term and singular boundary flux

$$\begin{cases} u_t = u_{xx} + (1-u)^{-p}, & 0 < x < 1,\ t > 0, \\ u_x(0,t) = 0,\ u_x(1,t) = (1-u(1,t))^{-q}, & t > 0, \\ u(x,0) = u_0(x), & 0 \le x \le 1. \end{cases} \tag{1.4}$$

Under some assumptions on initial data, they proved that quenching occurs in finite time and $x = 1$ is the only quenching point. Moreover, the lower bound of the quenching rate was estimated, i.e. $u(1,t) \ge 1 - C(T-t)^{1/(p+1)}$ if $p > 2q+1$ and $u(1,t) \ge 1 - C(T-t)^{1/[2(q+1)]}$ if $q \le p \le 2q+1$, as $t \to T^-$. However, they did not show the upper bound of the quenching rate.

To the best of our knowledge, very few works are concerned with the quenching rate of the quasilinear equations of p-Laplacian type, except for [13]. More precise, based on the work [2], Yang, Yin and Jin studied the p-Laplacian problem

$$\begin{cases} u_t = \left(|u_x|^{p-2} u_x\right)_x, & 0 < x < 1,\ t > 0, \\ u_x(0,t) = 0,\ u_x(1,t) = -g(u(1,t)), & t > 0, \\ u(x,0) = u_0(x), & 0 \le x \le 1, \end{cases} \tag{1.5}$$

where $\lim_{s \to 0^+} g(s) = +\infty$ and $g(s) > 0,\ g'(s) < 0$ for $s > 0$. They showed that $x = 1$ is the unique quenching point, and gave the quenching rate

$$\int_0^{u(1,t)} \frac{\mathrm{d}s}{-g^{p-1}(s) g'(s)} \sim C(T-t), \quad t \to T^-.$$

Later, Yang, Yin and Jin [14] studied the positive radial solutions to (1.5) in higher dimensional space and got the similar results to [13]. Besides, there are also some other singular properties for nonlinear parabolic equations such as L^∞ blowup and gradient blowup, see the latest papers [7,15,16,20,21] for examples and the references therein.

Motivated by the works [8,11,13,18], in this paper, we will study the quenching phenomenon of the more generalized equation (1.1). We prove that quenching occurs only at $x = 0$. Moreover, we give the bounds of the quenching rate and time. Our results are based on the ingenious construction of auxiliary functions. From our results, we know that the quenching rate of Problem (1.1) is really affected by both the reaction u^{-r} and the boundary flux u^{-q}.

Throughout this paper, we assume that the initial function u_0 satisfies

$$(|(u_0)_x|^{p-2}(u_0)_x)_x - u_0^{-r} \le 0, \text{ but } \not\equiv 0, \quad 0 \le x \le 1. \tag{1.6}$$

$$u_0 > 0, \quad (u_0)_x \ge 0 \text{ and } (u_0)_{xx} \le 0, \quad 0 \le x \le 1. \tag{1.7}$$

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