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Partial differential equations

A note on a global strong solution to the 2D Cauchy problem of density-dependent nematic liquid crystal flows with vacuum ☆

Sur la solution forte globale du problème de Cauchy pour l'écoulement d'un cristal liquide nématique bidimensionnel, dépendant de la densité et avec vide

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ABSTRACT

In Li–Liu–Zhong (*Nonlinearity* 30 (2017) 4062–4088), the authors proved the existence of a unique global strong solution to the Cauchy problem of 2D nonhomogeneous incompressible nematic liquid crystal flows with vacuum as far-field density provided the initial data density and the gradient of orientation decay not too slow at infinity, and the basic energy $\|\sqrt{\rho_0}\mathbf{u}_0\|_{L^2}^2 + \|\nabla \mathbf{d}_0\|_{L^2}^2$ is small. In this note, we aim at precisely describing this smallness condition.

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RÉSUMÉ

Dans Li–Liu–Zhong (*Nonlinearity* 30 (2017) 4062–4088), les auteurs démontrent l'existence d'une unique solution forte globale au problème de Cauchy pour l'écoulement d'un cristal liquide nématique, incompressible, non homogène, bidimensionnel, avec vide. Ce résultat est valide dans la mesure où la densité initiale donnée et le gradient de dérive d'orientation ne sont pas trop lents à l'infini et l'énergie de base $\|\sqrt{\rho_0}\mathbf{u}_0\|_{L^2}^2 + \|\nabla \mathbf{d}_0\|_{L^2}^2$ est petite. Le but de la présente Note est d'expliciter précisément cette dernière condition de petitesse.

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1. Introduction

In this note, we are concerned with the global existence of solutions to the following two-dimensional (2D) simplified version of nematic liquid crystal flows in the whole space $\mathbb{R}^2$, which describes the motion of a nonhomogeneous incompressible flow of nematic liquid crystals:

$$\begin{cases} \rho_t + \operatorname{div}(\rho\mathbf{u}) = 0, \\ (\rho\mathbf{u})_t + \operatorname{div}(\rho\mathbf{u}\otimes\mathbf{u}) - \Delta\mathbf{u} + \nabla P = -\operatorname{div}(\nabla\mathbf{d}\odot\nabla\mathbf{d}), \\ \mathbf{d}_t + (\mathbf{u}\cdot\nabla)\mathbf{d} = \Delta\mathbf{d} + |\nabla\mathbf{d}|^2\mathbf{d}, \\ \operatorname{div}\mathbf{u} = 0, \quad |\mathbf{d}| = 1 \end{cases} \tag{1.1}$$

together with the initial conditions

$$\rho(x,0) = \rho_0(x), \quad \rho\mathbf{u}(x,0) = \rho_0\mathbf{u}_0(x), \quad \mathbf{d}(x,0) = \mathbf{d}_0(x), \quad |\mathbf{d}_0(x)| = 1, \quad \text{in } \mathbb{R}^2. \tag{1.2}$$

Here, the unknown functions $\rho = \rho(x,t)$, $\mathbf{u} = (u_1,u_2)(x,t)$, and $P = P(x,t)$ denote the density, velocity, and pressure of the fluid, respectively. $\mathbf{d} = (d_1,d_2,d_3)(x,t)$ is the unknown (averaged) macroscopic/continuum molecule orientation of the nematic liquid crystal flow. The notation $\nabla\mathbf{d}\odot\nabla\mathbf{d}$ denotes the 2×2 matrix whose (i,j)-th entry is given by $\partial_i\mathbf{d}\cdot\partial_j\mathbf{d}$ $(1 \le i,j \le 2)$. The above system (1.1)–(1.2) is a macroscopic continuum description of the evolution for the nematic liquid crystals. It is a simplified version of the Ericksen–Leslie model [1,2], but it still retains most important mathematical structures as well as most of the essential difficulties of the original Ericksen–Leslie model. Some important progress has been made about the existence of weak and strong solutions to incompressible nematic liquid crystals equations for either homogeneous or nonhomogeneous fluids by many authors – refer to [4–7] and references therein.

Recently, Li–Liu–Zhong [3] established the global-in-time existence of a unique strong solution to the Cauchy problem (1.1)–(1.2) provided the initial data density and the gradient of orientation decay not too slow at infinity, and the basic energy $\|\sqrt{\rho_0}\mathbf{u}_0\|_{L^2}^2 + \|\nabla\mathbf{d}_0\|_{L^2}^2$ is small. Precisely, they showed the following result.

Theorem 1.1. *For constants $q > 2$, $a > 1$, assume that the initial data $(\rho_0, \mathbf{u}_0, \mathbf{d}_0)$ satisfies*

$$\begin{cases} \rho_0 \ge 0, \quad \rho_0\bar{x}^a \in L^1(\mathbb{R}^2)\cap H^1(\mathbb{R}^2)\cap W^{1,q}(\mathbb{R}^2), \quad \sqrt{\rho_0}\mathbf{u}_0 \in L^2(\mathbb{R}^2), \quad \nabla\mathbf{u}_0 \in L^2(\mathbb{R}^2), \\ \operatorname{div}\mathbf{u}_0 = 0, \quad \mathbf{d}_0 \in L^2(\mathbb{R}^2), \quad \nabla\mathbf{d}_0\bar{x}^{\frac{a}{2}} \in L^2(\mathbb{R}^2), \quad \nabla^2\mathbf{d}_0 \in L^2(\mathbb{R}^2), \quad |\mathbf{d}_0| = 1, \end{cases} \tag{1.3}$$

where

$$\bar{x} \triangleq (\mathrm{e} + |x|^2)^{\frac{1}{2}}\log^2(\mathrm{e} + |x|^2).$$

Then there is a positive constant ε_0 such that if

$$C_0 \triangleq \|\sqrt{\rho_0}\mathbf{u}_0\|_{L^2}^2 + \|\nabla\mathbf{d}_0\|_{L^2}^2 < \varepsilon_0, \tag{1.4}$$

then the Cauchy problem (1.1)–(1.2) has a unique global strong solution $(\rho, \mathbf{u}, P, \mathbf{d})$ satisfying that for any $0 < T < \infty$,

$$\begin{cases} 0 \le \rho \in C([0,T]; L^1\cap H^1\cap W^{1,q}), \\ \rho\bar{x}^a \in L^\infty(0,T; L^1\cap H^1\cap W^{1,q}) \\ \sqrt{\rho}\mathbf{u}, \nabla\mathbf{u}, \sqrt{t}\nabla\mathbf{u}, \sqrt{t}\sqrt{\rho}\mathbf{u}, \sqrt{t}\nabla P, t\nabla P, \sqrt{t}\nabla^2\mathbf{u}, t\nabla^2\mathbf{u} \in L^\infty(0,T;L^2), \\ \nabla\mathbf{d}, \nabla\mathbf{d}\bar{x}^{\frac{a}{2}}, \nabla^2\mathbf{d}, \sqrt{t}\nabla^2\mathbf{d}, \sqrt{t}\nabla\mathbf{d}_t, \sqrt{t}\nabla^3\mathbf{d}, t\nabla^3\mathbf{d} \in L^\infty(0,T;L^2), \\ \nabla\mathbf{u} \in L^2(0,T;H^1)\cap L^{\frac{q+1}{q}}(0,T;W^{1,q}), \\ \nabla P \in L^2(0,T;L^2)\cap L^{\frac{q+1}{q}}(0,T;L^q), \\ \nabla^2\mathbf{d} \in L^2(0,T;H^1), \quad \nabla\mathbf{d}_t, \nabla^2\mathbf{d}\bar{x}^{\frac{a}{2}} \in L^2(\mathbb{R}^2\times(0,T)), \\ \sqrt{\rho}\mathbf{u}_t, \sqrt{t}\nabla\mathbf{u}_t, \sqrt{t}\nabla\mathbf{d}_t, \sqrt{t}\nabla^2\mathbf{d}_t \in L^2(\mathbb{R}^2\times(0,T)), \\ \sqrt{t}\nabla\mathbf{u} \in L^2(0,T;W^{1,q}). \end{cases} \tag{1.5}$$

Moreover, the solution $(\rho, \mathbf{u}, P, \mathbf{d})$ has the following temporal decay rates, i.e., for all $t \ge 1$,

$$\|\nabla\mathbf{u}(\cdot,t)\|_{L^2}^2 + \|\nabla^2\mathbf{u}(\cdot,t)\|_{L^2} + \|\nabla P(\cdot,t)\|_{L^2} + \||\nabla\mathbf{d}||\nabla^2\mathbf{d}|(\cdot,t)\|_{L^2} + \|\nabla^2\mathbf{d}(\cdot,t)\|_{L^2}^2 \le Ct^{-1}, \tag{1.6}$$

where C depends only on C_0, $\|\rho_0\|_{L^1\cap L^\infty}$, and $\|\nabla\mathbf{u}_0\|_{L^2}$.

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