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Can libration maintain Enceladus's ocean?

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ABSTRACT

The process by which the subsurface ocean on Enceladus is heated remains a puzzle. Tidal interaction with Saturn and Dione is the leading candidate but whether the dominant heating occurs in the solid core, ice crust or in the ocean itself is an outstanding question. Here we consider the driving effect of the longitudinal libration of the ice crust on the subsurface ocean and argue that the flow response should be turbulent even in the most benign situation of a smooth spherical ice shell when the motion of the boundary is only transmitted viscously. A rigorous upper bound on the turbulent viscous dissipation rate is then derived and used to argue that libration should be potent enough to explain the observed heat flux emanating out of Enceladus when the effects of tidal distortion and roughness of the ice crust are included.

1. Introduction

The data from recent solar system probes suggest the existence of subsurface oceans in many icy satellites (Nimmo and Pappalardo, 2016) which opens up the fascinating possibility of extraterrestrial life relatively close by. Of the many oceans currently believed to exist, those in Europa, Titan and Enceladus are the most certain having been inferred by more than one type of measurement (e.g. for Enceladus see Thomas et al., 2016). Given the surface temperatures in the outer solar system, there is a question as to how these oceans avoid freezing and answering this is nowhere more challenging than on Enceladus given its small size (its radius is just 252 km) (Hussmann et al., 2006; Meyer and Wisdom, 2007; Roberts and Nimmo, 2008; Travis and Schubert, 2015; Nimmo and Pappalardo, 2016; Thomas et al., 2016). Cassini observed a heat flux of 15.8 ± 3.1 GW (Howett et al., 2011) emanating from Enceladus' south pole whereas radiogenic heating is only estimated to provide 0.3 GW (Schubert et al., 2007). Other possible explanations such as accretional heating, heat release through differentiation and exothermal chemical reactions have all been dismissed as insignificant leaving only tidal heating as a possible explanation (Hussmann et al., 2006; Travis and Schubert, 2015; Nimmo and Pappalardo, 2016). Enceladus is currently in a 2:1 orbital resonance with Dione and has an orbital eccentricity of 0.0047 with current observations suggesting that Enceladus's ice crust is decoupled from its rocky core: i.e. there is a global subsurface ocean (Schubert et al., 2007; Spencer and Nimmo, 2013; Thomas et al., 2016). The question is then where the dominant tidal heating occurs – is it in the rocky core, the ice-crust or in the ocean itself?

The ability of tidal effects to heat solids by the time-dependent distortions they induce is well understood and established after the successful prediction of Io's volcanic state (Peale et al., 1979; Morabito et al., 1979). As a result, the majority of studies have focussed either on Enceladus's solid core (Peale et al., 1980; Yoder, 1981; Poirier et al., 1983; Ross and Schubert, 1989; Choblet et al., 2017) or, more recently, on its ice crust (Roberts and Nimmo, 2008; Behounkova et al., 2013; Travis and Schubert, 2015). Despite this work, no generally-accepted solid model for tidally heating Enceladus's ocean has emerged (Nimmo and Pappalardo, 2016).

In contrast, the ability of tidal effects to drive fluid flows and thereby heat the ocean itself through the accompanying viscous dissipation is poorly understood. Dissecting the exact tidal response of the coupled system of solid core, subsurface ocean and overlying ice crust is a complicated problem fraught with unknowns. As a result, limiting scenarios have been studied in which a) the ice crust is a passive flexible skin over a tidal ocean and the counter scenario b) where the ice crust is rigid enough to suppress ocean tides but which instead drives the ocean through its libration. An attraction of the latter is that it replaces the tidal body force on the ocean by a boundary condition forcing which is conducive to laboratory modelling. The former limiting scenario was first considered by Tyler (2008, 2009, 2011, 2014) who used Laplace's 2D tidal equations for a shallow ocean to show via a resonant response that more than enough energy could be deposited in Enceladus's ocean given a large enough obliquity angle or a

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shallow enough ocean (see also Matsuyama, 2014; Hay and Matsuyama, 2017 and Chen et al., 2014 who claim that this is not the case for a non-resonant response).

Concurrently another body of work has considered the second limiting scenario by studying the effect of imposing a longitudinal libration on rotating fluid-filled spheres, spherical shells and ellipsoids (Noir et al., 2009; Calkins et al., 2010; Sauret et al., 2010; Cebron et al., 2012; Noir et al., 2012; Grannan et al., 2014). Here the approach has been a combination of laboratory and numerical experiments treating the full 3D Navier–Stokes equations. While the use of tidally-distorted containers is more realistic for planetary applications, librating axisymmetric containers are (far) easier to theoretically model and allow the simpler and weaker effect of only (viscous) tangential forcing to be understood. In this vein, the experiments of Noir et al. (2009) librate only spherical shells yet even then the flow clearly becomes turbulent near the boundary at sufficient librational amplitude. Later numerical work (Sauret et al., 2013) showed that inertial waves could be generated into the interior which, when driven at sufficient amplitude, should break down to small scale turbulence through triad resonances (Kerswell, 1999) as seen in experiments with ellipsoidal containers (Grannan et al., 2014). No estimates of the turbulent viscous heating have, however, emerged for either the axisymmetric or asymmetric container situations as yet due either to the difficulty of measuring it in the laboratory or resolving all the relevant length scales on the computer.

The purpose of this paper is remedy this situation at least in the axisymmetric situation by theoretically deriving a rigorous upper bound on the turbulent dissipation rate inside a librating fluid-filled spherical shell directly from the governing Navier–Stokes equations. The result makes a prediction for the scaling of the dissipation rate with the parameters of the problem (the libration amplitude, the libration frequency and Ekman number) in the limit of vanishing Ekman number. The numerical prefactor, however, is typically very conservative and data – either from laboratory experiments or numerical simulations – are needed to renormalise the bound down to make a realistic prediction of what dissipation actually occurs. Exactly this approach was used to estimate the turbulent dissipation rate possible in a precessing spheroid (Malkus, 1968; Kerswell, 1996). There the bound correctly predicts that the dissipation rate becomes independent of the precession rate and Ekman number but the numerical prefactor needs to be renormalised by one and a half orders of magnitude to match experimental data (see Fig. 1 in Kerswell, 1996). Significantly, even with this adjustment, this bound highlights the fact that precessionally-driven turbulence in the outer core is energetic enough to drive the geodynamo (Kerswell, 1996).

That the response to Enceladus's (longitudinal) libration should be turbulent even under the idealised assumption of a smooth axisymmetric ice crust–ocean interface is clear from experiments (Noir et al., 2009) and a simple estimate. The situation below the librating ice crust is essentially Stokes's second problem where an oscillating plane drives a half space of fluid. For Enceladus, the Reynolds number can be estimated as $Ro/\sqrt{E} \approx 7200$ where $Ro \approx 2.1 \times 10^{-3}$ is ratio of the peak librational rotation rate to the basic spin rate (the Rossby number) and the Ekman number $E \approx 8.5 \times 10^{-14}$ (see Table 1). This is well above the Reynolds number of ≈ 500 for which the flow to observed to be turbulent (Ozdemir et al., 2014).[1]

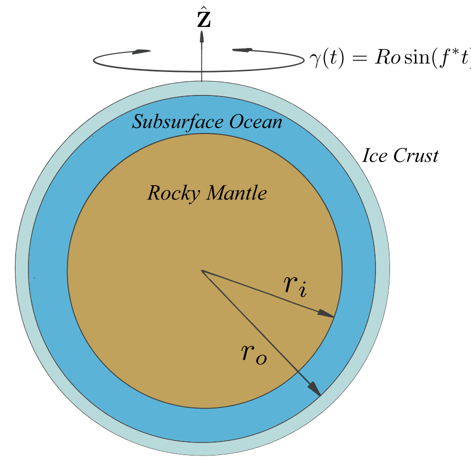

Fig. 1. A model of librating Enceladus. The picture is in the mantle frame which is presumed uniformly rotating. In this the crust is librating across the top of the subsurface ocean (inner radius r_i and outer radius r_o). Figure courtesy of Jerome Noir.

2. Bounding the dissipation in a librating spherical shell

Enceladus's subsurface ocean is modelled as a fluid-filled layer sandwiched between two concentric hard spheres of inner and outer radii $r = r_i$ and r_o respectively: see Fig. 1. For simplicity, the inner sphere is assumed to be uniformly rotating about its axis (labelled hereafter as the z-axis) at a constant angular frequency Ω (i.e. we ignore the libration of Enceladus's solid core which is estimated as being 4 times smaller than that of the ice crust: see Table 1 in Thomas et al., 2016) whilst the outer sphere is librating (so rotating about the same axis at $\Omega(1 + Ro\sin\omega t)$ where the Rossby number, Ro, is the amplitude (in units of the basic rotation rate Ω) and ω the angular frequency of the libration). Non-dimensionalising using the outer radius r_o and the base rotation rate Ω, the Navier–Stokes equations in the librating frame of the outer boundary[2] are then

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u}\cdot\nabla\boldsymbol{u} + 2[1+\gamma(t)]\hat{\boldsymbol{z}}\times\boldsymbol{u} + \dot{\gamma}(t)\hat{\boldsymbol{z}}\times\boldsymbol{r} + \nabla p = 2E\nabla\cdot\boldsymbol{e}, \quad (1)$$

where $\gamma(t) := Ro\sin(f^*t)$ with $f^* := \omega/\Omega$, $\dot{\gamma} := d\gamma/dt$, $e_{ij} := \frac{1}{2}(\partial_j u_i + \partial_i u_j)$, the Ekman number $E := \nu/r_o^2\Omega$ (ν is the kinematic viscosity) and the fluid is assumed incompressible

$$\nabla\cdot\boldsymbol{u} = 0. \quad (2)$$

In this librating frame the outer boundary is stationary so $\boldsymbol{u}|_{r=1} = \boldsymbol{0}$ while the inner boundary appears to oscillate back and forth with velocity $\boldsymbol{u}|_{r=1-d} = -\gamma(t)\hat{\boldsymbol{z}}\times\boldsymbol{r}$ where $d := (r_o - r_i)/r_o$ is the non-dimensional ocean depth. The (non-dimensionalised) long-time-averaged energy dissipation rate per unit mass is defined as

$$\varepsilon := 2E\langle \frac{1}{\mathcal{V}}\iiint_V e_{ij}e_{ij}dV \rangle \quad (3)$$

where $\mathcal{V}$ is the non-dimensional volume ($= 4/3\pi(1-(1-d)^3)$) and $\langle\ldots\rangle := \lim_{T\to\infty}\frac{1}{T}\int_0^T \ldots dt$ represents long-time averaging.

The method used to bound the dissipation requires taking 2 projections of equation (1) (Seis, 2015). The first is the energy equation, found by taking the long-time and volume averages of

[1] Note that the Reynolds number in Ozdemir et al. (2014) is $\sqrt{2}$ larger than the one used here.

[2] We use this frame to demonstrate the general situation although for the particular set up here, a frame fixed in the inner boundary would be easier to analyse.