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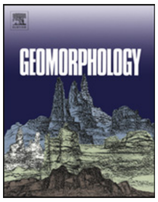

Uncertainty propagation in aeolian processes: From threshold shear velocity to sand transport rate

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ABSTRACT

The accurate estimation of aeolian saltation events is a fundamental requirement in the modelling of wind erosion, dust emission, dune movement and aeolian hazard prediction. A large number of semi-empirical sand transport rate models exist, with many relying on a single value for a shear velocity threshold above which saltation is initiated. However, measuring and modelling the sand transport rate suffers from the effects of a number of epistemic and aleatory uncertainties which make the identification of a single threshold value for shear velocity problematic. This paper focuses on the uncertainty propagation evident in calculations that use a threshold shear velocity to estimate sand transport rate. Probability density functions of threshold shear velocity are provided from the authors' previous studies. Grain diameter and shear velocity are considered as deterministically varying parameters. Several sand transport rate statistical metrics are estimated via the Monte Carlo approach adopting four different sand transport models. The sand transport rate estimation in probabilistic terms allows us to assess the amplification/reduction in the uncertainty and to provide a deeper insight into established transport rate models. We find that if the wind speed is close to the erosion threshold, every tested model amplifies the variability of the resulting estimated sand transport rate, especially in the case of coarse sand. If the wind speed is large, the adopted models present substantial differences in uncertainty. An interpretation of these differences is given by conditioning the sand transport rate models to the type of erosion threshold adopted, the fluid or impact threshold.

1. Introduction

The study of aeolian sand transport belongs to several research fields, from fundamental earth sciences to applied sciences such as civil and environmental engineering. From the scientific perspective, explaining and analysing windblown sand represent a challenging task due to the complex interactions between saltating particles, bed load and the wind field. Nevertheless, such analysis is an essential requirement in investigations of desert dust emissions (e.g. Haustein et al., 2015), dune dynamics (e.g. Wiggs and Weaver, 2012), agricultural wind erosion (e.g. Zobeck et al., 2003), land degradation (e.g. Mayaud et al., 2016), and planetary geomorphology (e.g. Kok et al., 2012). From the engineering perspective, windblown sand can have deleterious impacts on built structures and human activities (e.g. Xie et al., 2015; Zhang et al., 2010). For these reasons, the accurate prediction of sand transport events is a significant goal.

Saltation is the dominant mechanism of windblown sand transport. The total saltating load can be quantified by estimating the sand transport rate, i.e. by vertically integrating the horizontal flux of saltating particles. Since this physical quantity represents a straightforward measure to estimate wind erosion, sand transport, and deposition, a number of semi-empirical models to predict sand transport rate (*Q*-models) have been formulated (e.g. Kawamura, 1951; Kok et al., 2012; Lettau and Lettau, 1978; Owen, 1964).

Dong et al. (2003) classified sand transport models into four categories defined by their basic form. *Bagnold type* equations (e.g. Bagnold, 1941; Zingg, 1953) relate sand transport rate to the cube of shear velocity u_*^3 but do not explicitly consider the excess of shear velocity compared to a threshold value u_{*t}. This results in unrealistic sand transport rates when u_* is less than u_{*t}. *Modified Bagnold type* equations (e.g. Kawamura, 1951; Kok et al., 2012; Lettau and Lettau, 1978; Owen, 1964) relate sand transport rate to the cube of an effective shear velocity that is defined as a function of both

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the shear velocity and the threshold value. *O'Brien-Rindlaub type* and *modified O'Brien-Rindlaub type* equations (e.g. Dong et al., 2003; O'Brien and Rindlaub, 1936) relate transport rate to wind speed instead of shear velocity. These first three categories usually take into account the particle size directly through the sand grain diameter, *d*. The remaining models may be categorized as *complex*. These include physical models that account for additional phenomena in the saltation process such as inertial effects (Mayaud et al., 2017) or hysteresis (Kok, 2010). These models include multiple empirical fitting parameters usually related to quantities other than simply sand grain diameter.

Because of their ease of use and their sound physical basis, *modified Bagnold type* models are widespread in the literature and popularly employed in practice, see for example the field studies by Al-Awadhi and Al-Awadhi (2009), Barchyn and Hugenholtz (2011), Fryberger and Dean (1979), Liu et al. (2015), Sherman et al. (2013), Sherman and Li (2012), Yang et al. (2014). However, *modified Bagnold type* models lead to significant variability in their prediction, despite belonging to the same conceptual form (e.g. Sarre, 1987; Sherman et al., 2013, 1998; Sherman and Li, 2012). These discrepancies follow from differences in the structure of models and can be related to the way the effective shear velocity and the grain diameter are treated in the model. For example, whilst some models explicitly account for changes in *d* (e.g. Lettau and Lettau, 1978), others do not (e.g. Kawamura, 1951), and still others account for the effect of *d* by introducing other related variables, such as the particle terminal velocity in the model of Owen (1964).

These differences can be regarded as the result of the inherent *uncertainty* in the saltation phenomenon. To our knowledge, a comprehensive description of uncertainties concerning the prediction of aeolian sand transport rate is not available in the literature. A useful approach is to consider a general classification of uncertainty in sand transport rate predictions that distinguishes between *aleatory* and *epistemic* uncertainty (Zio and Pedroni, 2013), both of which are relevant to the sand transport case.

Aleatory uncertainty refers to the inherent randomness in many physical phenomena (e.g. Sørensen, 1993). It arises not only in nature but also in the laboratory environment, where the properties of aeolian processes can be nominally controlled in both space and time.

Epistemic uncertainty is associated with the lack of knowledge about the properties and conditions of the phenomena to be modeled, i.e. *model*, *measurement* and *parameter* uncertainties (see Barchyn et al., 2014; Shao, 2008). We believe that the uncertainty concerning the mode of u_{*t} to be used in sand transport equations can be considered as an *epistemic model* uncertainty too because it is related to the lack of knowledge about the *Q*-model. Indeed, the mode of u_{*t} to be adopted is not unequivocally established in the literature. Two threshold velocities have been recognized: the fluid (or static) threshold, i.e. the minimum wind speed for initiation of sediment transport without antecedent transport; and the impact (or dynamic) threshold, i.e. the minimum wind speed for sustaining sediment transport with antecedent transport. There is no unanimity in the literature as to which threshold is the most appropriate for modelling sand transport rate: some authors prefer the impact threshold, others suggest the fluid threshold, and still others recommend a combination of the two. Pye and Tsoar (2009) and Kok et al. (2012) recommend the impact threshold defined as a linear function of the fluid threshold (85% and 80% of the fluid threshold, respectively). Similarly, Andreotti (2004) and Pahtz et al. (2012) also prefer the impact threshold and provide models for its estimation. Conversely, Shao (2008) refers to the fluid threshold only, whilst Sherman et al. (2013) adopt the fluid threshold for small *Q* and, for increasing *Q*, an exponential decreasing u_{*t} to a minimum equal to the impact threshold (85% of the fluid threshold). Kok (2010) provides a more sophisticated model for sand transport which considers a hysteretic threshold between the impact and fluid threshold that depends on the history of the system.

The uncertainties reviewed up to this point are innate in *Q*-models. We expect that the *uncertainty propagation* to *Q* from other models also occurs, also due to the uncertainty in u_{*t}. A few authors have recently raised this issue. Shao (2008) attributes the *Q*-model randomness not only to their empirical parameters but also to variability in the threshold shear velocity. Moreover, since a method to determine a single quantitative definition of u_{*t} is not agreed upon (see Stout, 2004), Shao (2008) notes that any estimate of u_{*t} must involve a degree of subjectivity. In particular, he conjectured that such uncertainties in defining u_{*t} could outweigh the differences inherent in the functional forms of the sand transport rate models. The quantification of uncertainty in u_{*t} has recently been assessed by Edwards and Namikas (2015), Raffaele et al. (2016) and Webb et al. (2016) note that such uncertainty in threshold estimates can be expected to propagate to sand transport rate predictions.

Given these points, two main questions are pertinent: i. How does the degree of uncertainty in sand transport rate (*Q*) vary with respect to the uncertainty in estimates for the threshold shear velocity (u_{*t})? ii. How do different sand transport rate models behave when threshold shear velocity is considered as a statistically random variable?

The present study aims to contribute to a solution to these issues. Four key, semi-empirical models of sand transport rate are adopted to evaluate the impact of uncertainty propagation. Threshold shear velocity is assumed as the only random variable affecting sand transport rate and, as a result, instead of having a single deterministic value of sand transport rate for given values of u_* and *d*, a range of different values describing a probability distribution are obtained.

2. Methods

Here we describe the method for evaluating uncertainty propagation from the parametric uncertainty of the threshold shear velocity to the model prediction of sand transport rate. First, the general approach is described and justified. Secondly, the adopted sand transport rate models and threshold shear velocity probability density functions are given. In this and the following sections, the threshold shear velocity conditional probability density function $f(u_{*t} \mid d)$ is expressed as $f_{u_{*t}}$ for the sake of conciseness.

Uncertainty propagation from threshold shear velocity to predictions of sand transport rate is investigated by comparing dimensionless statistical metrics of both *Q* and u_{*t}. Both numerical and analytical solutions could be applied to evaluate uncertainty propagation (Smith, 2014). Analytically, for a given grain diameter and shear velocity, the cumulative distribution functions F_Q for sand transport rate can be obtained from the following procedure:

$$\begin{aligned} F_Q(s) &= P[Q \le s] = P[Q(u_{*t}) \le s] = P\left[u_{*t} \le Q^{-1}(s)\right] \\ &= F_{u_{*t}}\left[Q^{-1}(s)\right], \quad \forall d, u_* \end{aligned} \tag{1}$$

So, deriving each term, one can find the probability density functions f_Q:

$$f_Q(s) = f_{u_{*t}}\left[Q^{-1}(s)\right] \cdot \left[Q^{-1}(s)\right]', \quad \forall d, u_* \tag{2}$$

It is worth noting from Eq. (2) that the inversion of most of the sand transport rate models can only be performed numerically. Hence, we prefer a numerical approach because a fully analytical solution is not achievable. A classical Monte Carlo (MC) sampling