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Accurate volume integral solutions of direct current resistivity potentials for inhomogeneous conductivities in half space

Zhengyong Ren [a,b,c], Huang Chen [a], Jingtian Tang [a,b,c,*]

[a] School of Geosciences and Info-Physics, Central South University, 410083 Changsha, China
[b] Key Laboratory of Metallogenic Prediction of Nonferrous Metals and Geological Environment Monitoring, Ministry of Education, Central South University, 410083 Changsha, China
[c] Key Laboratory of Non-ferrous Resources and Geological Hazard Detection, 410083 Changsha, China

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ABSTRACT

We develop a novel accurate volume integral formula for solving potentials of three dimensional (3D) direct current resistivity problems with inhomogeneous conductivities in half-space. This new integral formula is composed of the potential, the gradient of Green's function, the gradient of the potential and the anomalous conductivity as the physical variables. First, the unstructured grids are adopted to handle inhomogeneous bodies with complicated shapes. Then, in each anomalous tetrahedron, the potential is represented as its values at vertices and the linear shape functions. Analytical expressions are developed to evaluate singular volume integrals in the final system of linear equations when the observation sites locate in the anomalous body. Finally, two synthetic models are utilized to verify the accuracy and convergence rate of our newly developed volume integral formula and its capability of dealing with complicated models with high conductivity contrasts.

1. Introduction

Direct current (DC) resistivity exploration technique has played a vital role in uncovering the conductivity structures of shallow subsurface. Static currents are injected into underground geological structures through source electrodes. Then, voltages are measured at potential electrodes. Using different configurations such as pole-pole, dipole-dipole configurations (Loke, 2000), geophysicists can calculate the apparent resistivity or use inversion routines to qualitatively or quantitatively interpret the conductivity distributions of the shallow Earth (Günther et al., 2006a; Seidel and Lange, 2007). Due to low cost and high resolution, it has been widely used in different geological and geophysical problems such as engineering and environment geophysics (Hauck et al., 2003; Kalscheuer et al., 2010; Rucker et al., 2010; Demirci et al., 2012), hydro-geophysical cases (Mansoor and Slater, 2007; Coscia et al., 2011; Doetsch et al., 2012; Doetsch et al., 2013;), archeological problems (Griffiths and Barker, 1994; Tsokas et al., 2008) and mineral explorations (Gochioco and Urosevic, 2003; Rucker, 2010; Yi et al., 2011).

To quantitatively invert the underground structures, DC resistivity methods need a reliable forward modeling solver with a task to compute the voltages at measuring sites for a given conductivity distributions underground. Unfortunately, analytical expression only exists for simple conductivity structure. For models with complicated conductivity distributions, we have to use numerical methods to conduct the forward modeling. At present, there are two principal sorts of numerical methods available for DC resistivity forward modeling, which are the differential methods, such as finite-element methods (Coggon, 1971; Bing and Greenhalgh, 2001; Li and Spitzer, 2002, 2005; Wu, 2003; Günther et al., 2006b; Zhou et al., 2009; Ren and Tang, 2010, 2014; Rücker, 2011; Udphuay et al., 2011), finite-difference (volume) methods (Dey and Morrison, 1979; Zhang et al., 1994; Spitzer, 1995; Loke and Barker, 1996; Wang et al., 2000), and the integral methods. The differential approaches have the capability of dealing with arbitrarily complicated conductivity structures and now have been largely adopted in geophysical communities. In contrast, the integral approaches only need to discretize the anomalous conductivity structures instead of the entire underground conductivity structures. In addition, the integral methods generally are based on semi-analytical formulae. Its solutions offer highly accurate solutions than other differential methods. As the integral solutions can be taken as reference solutions to verify the differential methods, therefore, it is still of great values to develop more accurate integral solutions.

Over the past five decades, three types of integral formulae were developed for DC resistivity problems, one kind of integral formula using charge-density as the physical variable (Alfano, 1959; Dieter et al., 1969; Pratt, 1972; Snyder, 1976; Daniels, 1977; Spiegel et al., 1980; Eskola et al., 1984; Das and Parasnis, 1987; Li and Oldenburg, 1991;

* Corresponding author at: School of Geosciences and Info-Physics, Central South University, 410083 Changsha, China.
E-mail address: jttang@csu.edu.cn (J. Tang).

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Eskola and Hongisto, 1997; Boulanger and Chouteau, 2005), one kind of integral formula using electric field or current-density (Eskola, 1979; Okabe, 1981; Eloranta, 1984; Eloranta, 1988; Beard et al., 1996; Li and Uren, 1997; Méndezdelgado et al., 1999) and the one using potential (Phillips, 1934; Lee, 1975; Hvoždara and Petr, 1982; Hvoždara and Petr, 1983; Eloranta, 1986; Poirmeur and Vasseur, 1988; Hvoždara, 1994; Hvoždara and Kaikkonen, 1994, 1996; Hvoždara and Kaikkonen, 1998; Xu et al., 1998; Xu et al., 1988; Ma, 2002; Ciulli et al., 2004). The integral formula using potential as the physical variable has better stability, faster convergences and is more effective to deal with high conductivity contrast (Eloranta, 1986). However, most integral formulae using potential were expressed in form of surface integral formula. Although reduction from three dimensions to two dimensions can significantly reduce the computational cost, it is more valuable to develop volume integral formulae to solve complicated 3D DC resistivity problems due to its flexibility of dealing with complicated structures. The first attempt of 3D direct current resistivity volume integral formula was conducted by Hvoždara and Kaikkonen (1998) for the case of inhomogeneous conductivities buried in a half-space. Because the gradient of anomalous conductivity and the potential were involved in this volume integral formula, the constant approximation of the unknown potential in each element should be chosen and its singular volume integral was transformed into the surface integral which only can be evaluated using numerical quadrature approaches. They lead that the accuracies of numerical solutions were decreased.

To overcome the possible accuracy lose in the above volume integral formula for direct current resistivity modeling, in this study, we develop a new alternative volume integral formula in which the gradient of the anomalous conductivity is transferred onto the potential. Therefore, a constant conductivity value can be adopted in each element so that the singular Green's function volume integrals have simple expressions. In addition, we present a set of new closed-form solutions for these singular volume integrals over tetrahedral bodies which enhance the capability of our new algorithm to deal with complicated anomalous conductivity targets under the ground. Furthermore, linear shape functions are used to represent the unknown potentials which are assigned to nodes of the tetrahedral grids. Unlike the constant shape functions which were used in previous studies, employing of the linear shape functions can not only increase the numerical accuracy of solutions but also can dramatically reduce the computation cost on the same grid. It is because that the number of nodes is generally much less than the number of elements for a discretized grid of the anomalous conductivity targets.

Two synthetic complicated models are used to verify the accuracy of our newly developed volume integral formulae. Excellent agreements are obtained among our solutions and other published analytic solutions and finite-element numerical solutions.

2. Problem statement

2.1. Boundary value problem

The boundary value problem for total potential U in DC resistivity problems is defined as follows:

$$\nabla\cdot\sigma\nabla U=-2I\delta(\mathbf{r}-\mathbf{r}_A)\quad \text{in}\,\Omega, \tag{1}$$

$$\nabla U\cdot\hat{\mathbf{n}}=0\quad \text{on}\,\Gamma_0, \tag{2}$$

$$U=0\quad \text{on}\,\Gamma_1, \tag{3}$$

where σ is the conductivity in the earth Ω, $\mathbf{r}$is an arbitrary point in Ω, $\mathbf{r}_A$ denotes the source electrode where a direct current with an amplitude of I is injected. Symbol Γ_0 denotes the air-earth interface, Γ_1 is the infinite boundary and $\hat{\mathbf{n}}$ is the outgoing normal vector on surfaces $\Gamma_0\cup\Gamma_1=\partial\Omega$. The geoelectric configuration is shown in Fig. 1. In Fig. 1, a right-handed coordinate system is built up on $\Gamma_0(z=0)$ with positive z-axis downward. The abnormal regions are defined as $\Omega_a=\sum_{i=1}^{n}\Omega_i$ n is the number of abnormal bodies. The anomalous conductivity is σ_a. The background conductivity is denoted as σ_0. The conductivities of the anomalies are $\sigma_i=\sigma_0+\sigma_a$ $(i=1,\ldots,n)$.

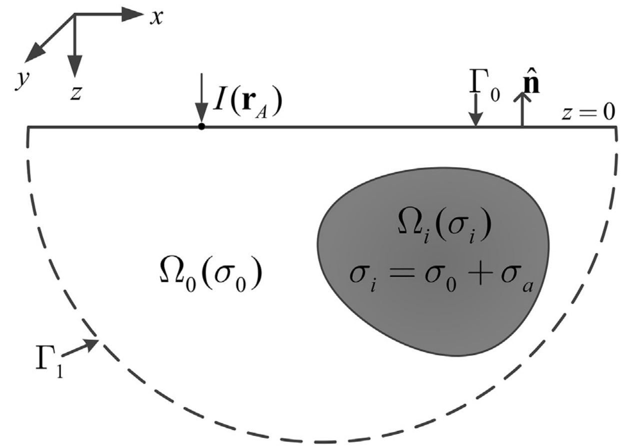

Fig. 1. Illustration of the geoelectric configuration. A right-handed Cartesian coordinate system is set up, the x- and y-axes are horizontal and the z-axis is downward with the origin on the ground surface. Symbol Ω denotes the Earth which consists of a background region Ω_0 with background conductivity σ_0 and a set of abnormal regions $\Omega_a=\sum_{i=1}^{n}\Omega_i$ with anomalous conductivity σ_a. σ_i denotes the conductivity in Ω_i, satisfying $\sigma_i=\sigma_0+\sigma_a$. Symbol Γ_0 denotes the air-earth interface, Γ_1 denotes the infinite boundary and $\hat{\mathbf{n}}$ is the outgoing normal vector on surfaces $\Gamma_0\cup\Gamma_1=\partial\Omega$. $\mathbf{r}_A$ denotes the source electrode where a direct current with an amplitude of $I$is injected.

Firstly, the total potential U is decomposed into two parts:

$$U=U_p+U_s, \tag{4}$$

where U_p denotes the primary potential generated by direct current I over the background model with conductivity distribution σ_0 and U_s is the secondary potential. The primary potential U_p satisfies:

$$\nabla\cdot\sigma_0\nabla U_p=-2I\delta(\mathbf{r}-\mathbf{r}_A)\quad \text{in}\,\Omega, \tag{5}$$

with boundary conditions:

$$\nabla U_p\cdot\hat{\mathbf{n}}=0\;\;\text{on}\;\;\Gamma_0, \tag{6}$$

$$U_p=0\quad \text{on}\,\Gamma_1. \tag{7}$$

Subtracting Eq. (1) by Eq. (5) and then substituting equation $\sigma=\sigma_0+\sigma_a$, we have:

$$\sigma_0\nabla^2(U-U_p)+\nabla\cdot\sigma_a\nabla U=0\quad \text{in}\,\Omega, \tag{8}$$

where $\sigma_a=0\;in\;\Omega_0$.

Substituting Eq. (4) into Eq. (8), we get:

$$\nabla^2U_s=-\frac{\nabla\cdot\sigma_a\nabla U}{\sigma_0}=-\frac{\nabla\cdot\mathbf{j}_a}{\sigma_0}\quad \text{in}\;\Omega.\theta \tag{9}$$

Using Eqs. (2)–(3) and Eqs. (6)–(7), we get:

$$\nabla U_s\cdot\hat{\mathbf{n}}=0\quad \text{on}\;\;\Gamma_0, \tag{10}$$

$$U_s=0\quad \text{on}\;\;\Gamma_1. \tag{11}$$